EE571
Solution to Prelab 1

1a) To find t_p, we need to take dy/dt and set equal to zero:

$$\frac{dy}{dt} = \zeta\omega_n e^{-\zeta\omega_n t}(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t) - e^{-\zeta w_n t}(-\omega_d \sin\omega_d t + \frac{\omega_d\zeta}{\sqrt{1-\zeta^2}}\cos\omega_d t) = 0$$

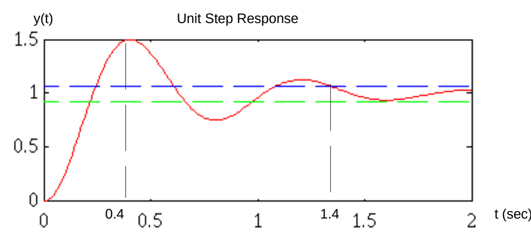
$$\Rightarrow (\zeta\omega_n - \frac{\omega_d\zeta}{\sqrt{1-\zeta^2}})\cos\omega_d t + (\frac{\omega_n\zeta^2}{\sqrt{1-\zeta^2}} - \omega_d)\sin\omega_d t = (\zeta\omega_n - \frac{\omega_n\zeta\sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}})\cos\omega_d t + (\frac{\omega_n\zeta^2}{\sqrt{1-\zeta^2}} - \omega_n\sqrt{1-\zeta^2})\sin\omega_d t = 0$$

$$\Rightarrow (\zeta\omega_n - \zeta\omega_n)\cos\omega_d t + (\frac{2\omega_n\zeta^2 - \omega_n}{\sqrt{1-\zeta^2}})\sin\omega_d t = 0 = \sin\omega_d t \quad \therefore \omega_p t_p = \sin^{-1}(0) = \pi \Rightarrow t_p = \pi/\omega_d = \pi/(\omega_n\sqrt{1-\zeta^2})$$

b) $$y(t_p) = 1 - e^{-\zeta\omega_n\pi/\omega_n\sqrt{1-\zeta^2}}(\cos\pi + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\pi) = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$\Rightarrow M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} = \frac{1-(1+e^{-\zeta\pi/\sqrt{1-\zeta^2}})}{1} = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$$

2. a) From the following plot:



t_p=0.4 seconds and M_p=0.5 x 100%=50%. Therefore, $\ln(M_p)=\ln(0.5)=-\zeta\pi/(1-\zeta)^{1/2} \Rightarrow \ln(0.5)^2=(\zeta\pi)^2/(1-\zeta)$
$\Rightarrow \zeta = [\ln(0.5)^2/(\pi^2+\ln(0.5)^2]^{1/2}=0.21545$

$\Rightarrow \omega_n = \pi/[t_p(1-\zeta^2)^{1/2}] = \pi/[0.4(1-(0.21545)^2)^{1/2}] = 8.04288$. Let's substitute these values to find the transfer function:

$$\frac{Y(s)}{W(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{64.6879}{s^2 + 3.46568s + 64.6879}$$

b) Note that $y(t_s)=1.02=1+e^{-\zeta\omega_n t_s}$. Therefore, $t_s=-1/\zeta\omega_n \times \ln(0.02) = 3.91/\zeta\omega_n$

c) From plot, t_s =1.4 seconds $\Rightarrow$ 1.4 = $3.91/\zeta\omega_n$ $\Rightarrow$ If ζ = 0.21545 then ω_n = 12.96961 which is close to the previous value of ω_n = 8.04288