Prelab is due Wednesday, Sept. 14 **EE571** Experiment #1

Objective: The objective of the first two labs is to learn a variety of system modelling techniques. In this first experiment, you will learn: 1) How to obtain a transfer function from a step response; 2) How to obtain a state variable model from a transfer function; 3) How to measure internal parameters of a system using voltage vs. time and voltage vs. voltage plots.

Prelab: Counts as HW#5 (due Wednesday, September 14):

All seven experiments in EE571 will use the MOTOMATIC DC servo as the system. **Figure 1** shows a schematic diagram for the MOTOMATIC.

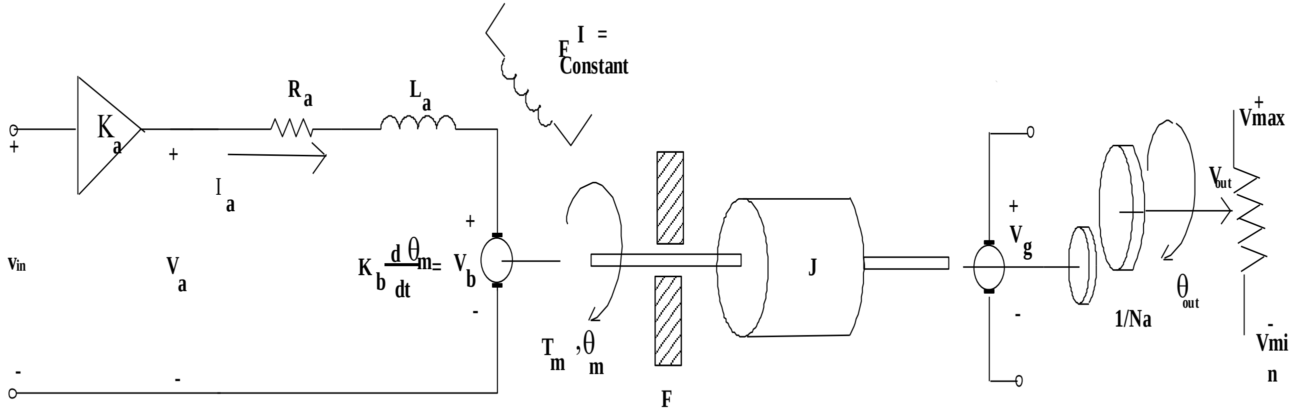

Figure 1. *Schematic of MOTOMATIC DC servo motor system.*

The MOTOMATIC is simply a DC servo with a voltage input (Vin) and two voltage outputs (Vout which is proportional to the angular displacement, θ_{out}, and Vg which is proportional to the speed of the motor, ω_m). In Experiment 1, you will obtain the transfer function, Vout(s)/Vin(s), from a step response plus you will measure the internal variables Kp (the potentiometer constant), Kg (the generator (tachometer) constant), and Kb (the back EMF constant). In Experiment 2, you will measure the remaining variables of the MOTOMATIC.

1. We learned in class that a classical second order system has the transfer function,

$$\frac{Y(s)}{W(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ζ is the damping coefficient and ω_n is the natural frequency. We also learned that the unit step response of such a system has the form:

$$y(t)|_{w(t)=u(t)} = 1 - e^{-\zeta w_n t}(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t)$$

where $\omega_d = \omega_n(1-\zeta^2)^{1/2}$ is the damped natural frequency and we have assumed an underdamped system (i.e., $\zeta < 1$)

a) Find an expression for the peak time (t_p) which is the time at which $y|_{step}$ is at a maximum (your answer should be in terms of ζ and ω_n .

b) Find an expression for $y|_{step}(t_p)$ then find an expression for the maximum percent overshoot

$$M_p = \left[\frac{y_{max} - y_{\infty}}{y_{\infty}} \right] \times 100\%$$

where y_{max} is the maximum value of y(t), and y_{∞} is the steady-state value of y(t).

2. a) From your answers to part 1a) and 1b), find the values of ζ and ω_n from the following plot:

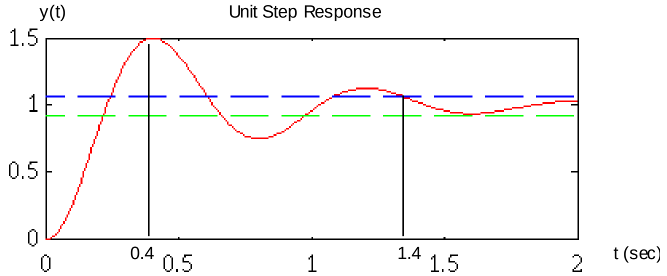

Figure 2. *Unit Step Response of an approximate 2nd-order system*

b) Another parameter that we can measure from the step response is the settling time, t_s. The settling time is defined as the time required for the system to settle within 2% of the steady-state value of y(t). If we assume that the term, $(\cos\omega_d t + \zeta/(1-\zeta^2)^{1/2}\sin\omega_d t)$ has a magnitude of 1, calculate the value of t_s in terms of ζ and ω_n.

c) Find an numerical value for t_s from the step response shown in part 2a). Then, use the value of ζ found in 2a) and your expression for t_s to find ω_n. Is this value close to your value ω_n found in part 2a)?

In Lab: Counts as HW#6 (Due Monday, September 19)

1. Obtain a closed loop step response of the MOTOMATIC when the input is 5u(t). Repeat problems 2a) and 2c) of the prelab but now use the MOTOMATIC step response. You should obtain two different answers for ω_n form the settling time and the peak time. Are these values similar? Why or why not? Find the transfer function, Y(s)/W(s) for both values of ω_n.

2. Another way to obtain a model for the system is to measure the system's internal parameters then, using the structure of the system, find a transfer function in terms of the systems parameters.

a) Vout is proportional to the output angular displacement, θ_{out} or we can say that Vout = $K_p\theta_{out}$ where K_p is the unknown constant of proportionality. Physically, Vout is the voltage from the wiper arm of a potentiometer which is coupled to the output shaft. The pot voltage varies linearly from Vmax to Vmin when the output shaft is turning at a constant speed. The slope of this linear region is K_p. Obtain a plot of Vout vs. time with the motor turning at a constant speed and find Vmax, Vmin, and K_p.

b) The MOTOMATIC has a speed reducer (gear train) attached to the motor such that the output shaft turns 9 times slower than the motor shaft. Both the generator constant (K_g) and the back EMF constant, K_b are measured BEFORE the speed reducer. The relationship for the unknown generator (tachometer) constant is $V_g = K_g\omega_m$ where ω_m is the MOTOR shaft speed. If we integrate the generator voltage we obtain:

$$\int V_g dt = K_g\theta_m = K_g(N_a \times \theta_{out}) = K_g \times N_a \times \frac{V_{out}}{K_p}$$

Obtain a plot of $\int V_g dt$ vs. Vout and find K_g from this plot.

c) Obtain a plot of the integral of the back EMF voltage, $\int V_b dt$ vs. Vout and find the back EMF constant from,

$$\int V_b dt = K_b \theta_m = K_b (N_a \times \theta_{out}) = K_b \times N_a \times \frac{V_{out}}{K_p}$$

Note: The Lab is in the back of room 551 of Anderson Hall (FPAT). There, you will find one MOTOMATIC station set-up to run experiment 1.