

0. Lab 1 (the in lab portion) is due Monday. Each person should e-mail his/her own lab.

1. Given the following state variable model (see HW#4):

$$\dot{x} = \begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w, \quad x(-3) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \end{bmatrix} w$$

a) Use the solution, $x(t) = e^{A(t-t_0)} x(t_0) + \int_{t_0}^t e^{A(t-\tau)} B w(\tau) d\tau$ derived last time to find $x(t)$ if the input to our system is $w(t)=e^{-t}u(t)$.

b) Use your solution to find $y(t)$

c) Assume that the initial condition is zero then find the transfer function $H(s)=Y(s)/W(s)$.

d) Determine the stability of $\dot{x} = Ax + Bw(t), x(t_0)$ if $A =$

i) $\begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$ ii) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ iii) $\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ iv) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

e) Determine the stability of the systems modelled by the following transfer functions

i) $H(s) = \frac{10(s-1)}{(s+1)(s+2)}$ ii) $H(s) = \frac{10(s-1)}{(s+1)(s-2)}$ iii) $H(s) = \frac{10(s-1)}{s(s+1)}$ iv) $H(s) = \frac{10(s-1)}{s^2(s+1)}$

f) Draw the overall block diagram for the following automatic shutter system. The system works by using a servomotor (similar to our Motomatic) and a worm gear to open a 2 meter by 2 meter square aperture. The worm gear converts angular displacement to linear displacement with a ratio of 100 radians = 1 meter. A photo cell is located on the exposed plate and converts light into a voltage. The amount of illumination the photocell is exposed to is 1 candela (cd) for every square meter of aperture opening. The photocell outputs a voltage of 1 volt per candela. This voltage is subtracted from a reference voltage by means of a differential amplifier. The motor torque constant is 0.1 Nm/A (i.e., $T_m = 0.1 I_a$) and the motor inertia is $J_a = 1 \text{ kgm}^2$. The viscous friction constant is $F=0.2\text{Nms}$, the winding resistance is $R_a=1 \text{ ohm}$ and the amplifier gain is $K_a=50$. The back emf constant is $K_b = .2 \text{ Vs/rad}$ and the armature inductance is $L_a = 100 \text{ mH}$.

