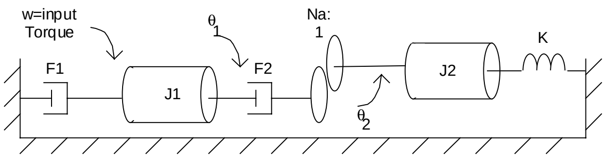
EE571 Solution to HW#3

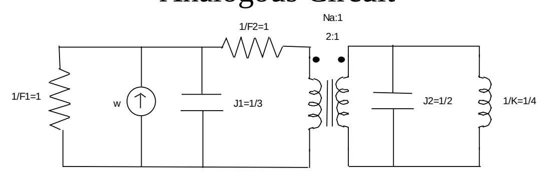
0. Have you picked a lab partner to work with you? Your first lab is coming soon!

1a) The analogous circuit for the first problem is:

Original Circuit



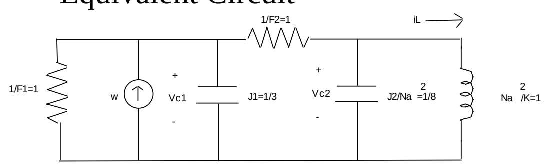

Analogous Circuit



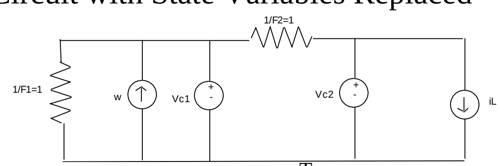

where F1=F2=1 Nms/rad, K= 4 Nm/rad, J1= 1/3 $Nms^2$,J2= 1/2 $Nms^2$ and Na=2

The admittance seen to the left of the transformer is Y=J2s+K/s. Therefore, this admittance reflected across the transformer is: $Y_{eq}=Y/Na^2=(J2s+K/s)/Na^2$. This reflected admittance would come from a capacitor of $J2/Na^2$ in parallel with an inductor with a value of $Na^2/K$ Thus, the equivalent circuit is given by:

Equivalent Circuit




Circuit with State Variables Replaced



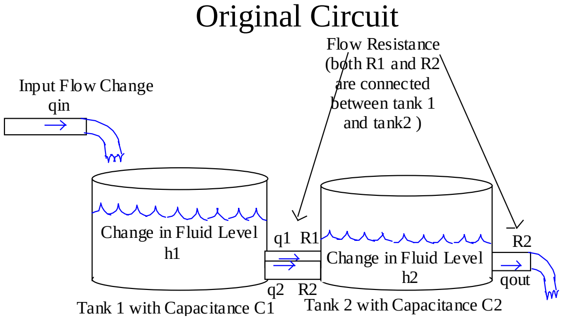

The state variable of the analogous electrical circuit is $x=[V_{c1}\ V_{c2}\ i_L\ ]^T$. Thus:

$$\dot{x}=\begin{bmatrix}\dot{V}_{c1}\\ \dot{V}_{c2}\\ i_L\end{bmatrix}=\begin{bmatrix}1/C_1(i_{c1})\\ 1/C_2(i_{c2})\\ 1/L(v_L)\end{bmatrix}=\begin{bmatrix}3(i_{c1})\\ 8(i_{c2})\\ 1(v_L)\end{bmatrix}=\begin{bmatrix}3(i_{c1}|_{V_{c1}}+i_{c1}|_{V_{c2}}+i_{c1}|_{i_L}+i_{c1}|_{w})\\ 8(i_{c2}|_{V_{c1}}+i_{c1}|_{V_{c2}}+i_{c2}|_{i_L}+i_{c2}|_{w})\\ 1(v_L|_{V_{c1}}+v_L|_{V_{c2}}+v_L|_{i_L}+v_L|_{w})\end{bmatrix}=\begin{bmatrix}3(-2v_{c1}+1v_{c2}+0i_L+1w)\\ 8(1v_{c1}-1v_{c2}-1i_L+0w)\\ 1(0v_{c1}+v_{c2}+0i_L+0w)\end{bmatrix}$$

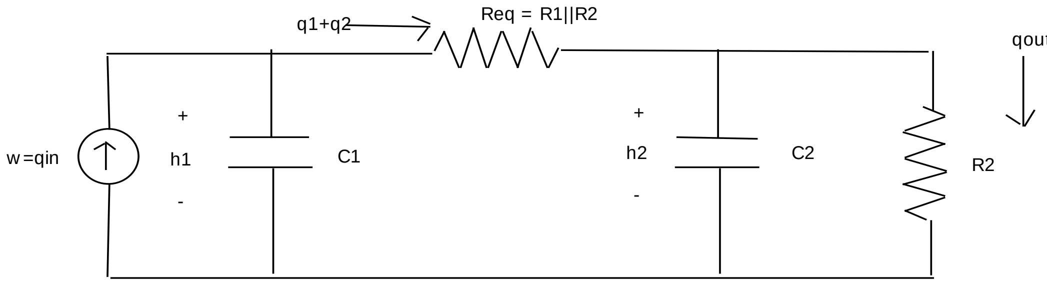
or $\dot{x}=\begin{bmatrix}-6 & 3 & 0\\ 8 & -8 & -8\\ 0 & 1 & 0\end{bmatrix}+\begin{bmatrix}3\\ 0\\ 0\end{bmatrix}w$

1b) The analogous circuit for problem 1b) is:

Original Circuit




Analogous Circuit




The state variable of the analogous electrical circuit is x=[$h_c$ $h_2$ ]$^T$. Thus:

$$\dot{x} = \begin{bmatrix}\dot{h}_1\\ \dot{h}_2\end{bmatrix} = \begin{bmatrix}1/C_1(i_{c1})\\ 1/C_2(i_{c2})\end{bmatrix} = \begin{bmatrix}1/C_1(i_{c1}\big|_{h_1} + i_{c1}\big|_{h_2} + i_{c1}\big|_{w})\\ 1/C_2(i_{c2}\big|_{h_1} + i_{c2}\big|_{h_2} + i_{c2}\big|_{w})\end{bmatrix} = \begin{bmatrix}1/C_1(-h_1/R_{eq} + h_2/R_2 + w)\\ 1/C_2(h_1/R_1 - h_2(1/R_{eq} + 1/R_2) + 0w)\end{bmatrix}$$

or $\dot{x} = \begin{bmatrix}\dfrac{-1}{R_{eq}C_1} & \dfrac{1}{R_{eq}C_1}\\ \dfrac{1}{R_{eq}C_2} & \dfrac{-R_{eq}R_2}{R_{eq}C_2 + R_2C_2}\end{bmatrix}x + \begin{bmatrix}1/C_1\\ 0\end{bmatrix}w$ where $R_{eq}= R_1R_2/(R_1+R_2)$

2a) i) First let's find the eigenvalues of A:

$$A = \begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \Rightarrow |sI - A| = \begin{vmatrix}s-2 & -1\\ -1 & s-2\end{vmatrix} = s^2 - 4s + 3 = (s-1)(s-3) = 0 \therefore s_1 = 1 \, and \, s_2 = 3$$

Next, let's find the eigenvectors of A:

$$[s_1I - A]\underline{P}_1 = \begin{bmatrix}1-2 & -1\\ -1 & 1-2\end{bmatrix}\begin{bmatrix}p_1\\ p_2\end{bmatrix} = \begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}\begin{bmatrix}p_{11}\\ p_{12}\end{bmatrix} = 0 \therefore \underline{P}_1 = \begin{bmatrix}p_{11}\\ p_{12}\end{bmatrix} = \begin{bmatrix}1\\ -1\end{bmatrix}$$

$$[s_2I - A]\underline{P}_2 = \begin{bmatrix}3-2 & -1\\ -1 & 3-2\end{bmatrix}\begin{bmatrix}p_1\\ p_2\end{bmatrix} = \begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}p_{21}\\ p_{22}\end{bmatrix} = 0 \therefore \underline{P}_2 = \begin{bmatrix}p_{21}\\ p_{22}\end{bmatrix} = \begin{bmatrix}1\\ 1\end{bmatrix}$$

ii) First let's find the eigenvalues of A:

$$A = \begin{bmatrix}0 & 2\\ 12 & 2\end{bmatrix} \Rightarrow |sI - A| = \begin{vmatrix}s & -2\\ -12 & s-2\end{vmatrix} = s^2 - 2s + -12 = (s-6)(s+4) = 0 \therefore s_1 = 6 \, and \, s_2 = -4$$

Next, let's find the eigenvectors of A:

$$[s_1I - A]\underline{P}_1 = \begin{bmatrix}6 & -2\\ -12 & 6-2\end{bmatrix}\begin{bmatrix}p_1\\ p_2\end{bmatrix} = \begin{bmatrix}6 & -2\\ -12 & 4\end{bmatrix}\begin{bmatrix}p_{11}\\ p_{12}\end{bmatrix} = 0 \therefore \underline{P}_1 = \begin{bmatrix}p_{11}\\ p_{12}\end{bmatrix} = \begin{bmatrix}1\\ 3\end{bmatrix}$$

$$[s_2I - A]\underline{P}_2 = \begin{bmatrix}-4 & -2\\ -12 & -4-2\end{bmatrix}\begin{bmatrix}p_1\\ p_2\end{bmatrix} = \begin{bmatrix}-4 & -2\\ -12 & -6\end{bmatrix}\begin{bmatrix}p_{21}\\ p_{22}\end{bmatrix} = 0 \therefore \underline{P}_2 = \begin{bmatrix}p_{21}\\ p_{22}\end{bmatrix} = \begin{bmatrix}1\\ -2\end{bmatrix}$$

iii) First let's find the eigenvalues of A:

$$A = \begin{bmatrix} 0 & 2 \\ -16 & -8 \end{bmatrix} \Rightarrow |sI - A| = \begin{vmatrix} s & -2 \\ 16 & s+8 \end{vmatrix} = s^2 + 8s + 16 = (s + 4 + j4)(s + 4 - j4) = 0 \therefore s_1 = -4 - j4 \, and \, s_2 = -4 + j4$$

Next, let's find the eigenvectors of A:

$$[s_1 I - A]\underline{P}_1 = \begin{bmatrix} -4 - j4 & -2 \\ 16 & -4 - j4 + 8 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} -4 - j4 & -2 \\ 16 & 4 - j4 \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix} = 0 \therefore \underline{P}_1 = \begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 - j2 \end{bmatrix}$$

$$[s_2 I - A]\underline{P}_2 = \begin{bmatrix} -4 + j4 & -2 \\ 16 & -4 + j4 + 8 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} -4 - j4 & -2 \\ 16 & 4 + j4 \end{bmatrix} \begin{bmatrix} p_{21} \\ p_{22} \end{bmatrix} = 0 \therefore \underline{P}_2 = \begin{bmatrix} p_{21} \\ p_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 + j2 \end{bmatrix}$$