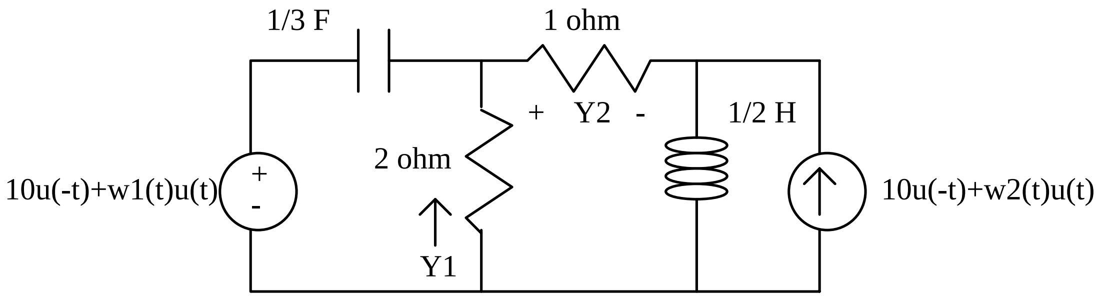
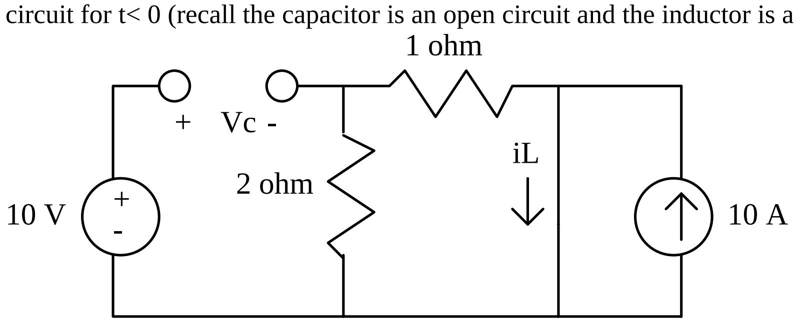
EE571 - Solution to HW#2

1. a) Find the state variable model of the form $\begin{array}{l}\dot{x} = Ax + Bw,\ x(0^+)\\ y = Cx + Dw\end{array}$ for the following electrical network:

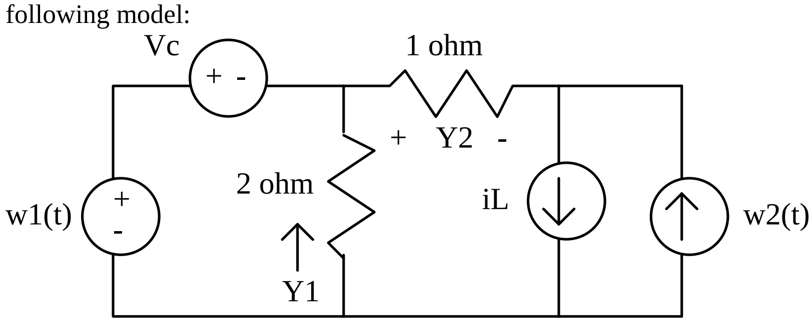


(make sure to include your initial conditions)

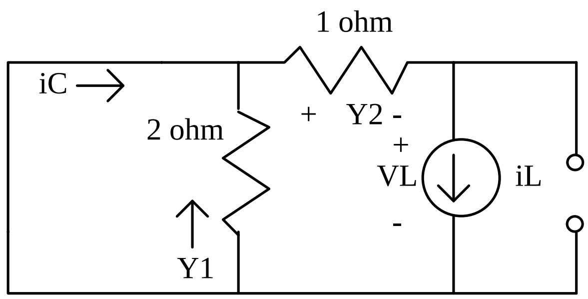
Note that there are no redundant state variables! Let $x = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$. To find our initial conditions, let us redraw the circuit for t< 0 (recall the capacitor is an open circuit and the inductor is a short circuit for steady-state DC:



Obviously, $V_c(0^-)$ = 10 volts while $i_L(0^-)$ = 10 Amps. To obtain the complete state variable model, first note that $\dot{x} = \begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} \frac{1}{L} v_L \\ \frac{1}{C} i_C \end{bmatrix} = \begin{bmatrix} \frac{1}{L}(v_L|_{i_L} + v_L|_{v_c} + v_L|_{w_1} + v_L|_{w_2}) \\ \frac{1}{C}(i_C|_{i_L} + i_C|_{v_C} + i_C|_{w_1} + i_C|_{w_2}) \end{bmatrix}$. Similarly, the output equation can be expressed as $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} (y_1|_{i_L} + y_1|_{v_c} + y_1|_{w_1} + y_1|_{w_2}) \\ (y_2|_{i_L} + y_2|_{v_C} + y_2|_{w_1} + y_2|_{w_2}) \end{bmatrix}$. Our task now is to express the right hand side of both the state and output equations solely in terms of state variables (i_L and v_c) and inputs (w_1 and w_2). To this end, let us use our trick of replacing the capacitors by voltage sources and inductors by current sources to produce the following model:

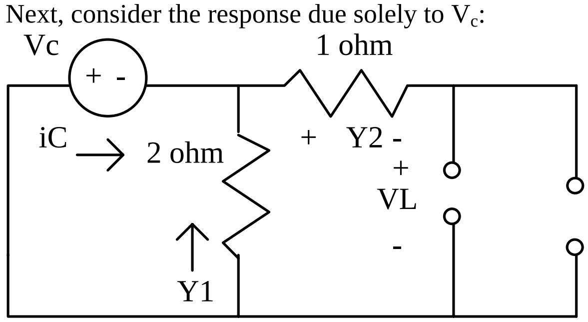


Note that by employing this trick, ALL responses of the circuit will be in terms of state variables (i_L and v_c) and inputs (w_1 and w_2). By utilizing superposition, we can solve for the contribution of each source individually:
First, let's consider the response of the circuit just due to i_L:



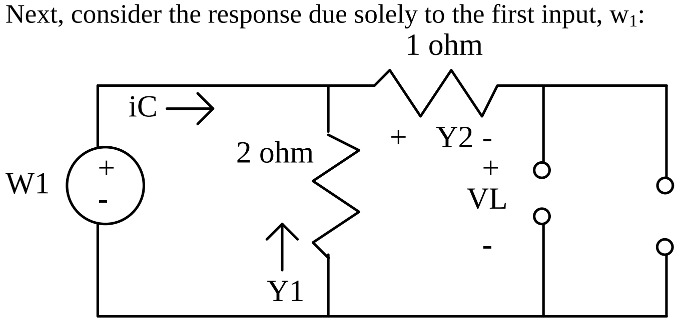
Clearly, $V_L|_{iL} = -1i_L$, $i_C|_{iL} = 1i_L$, $Y_1|_{iL} = 0i_L$, and $Y_2|_{iL} = 1i_L$.

Next, consider the response due solely to V_c:



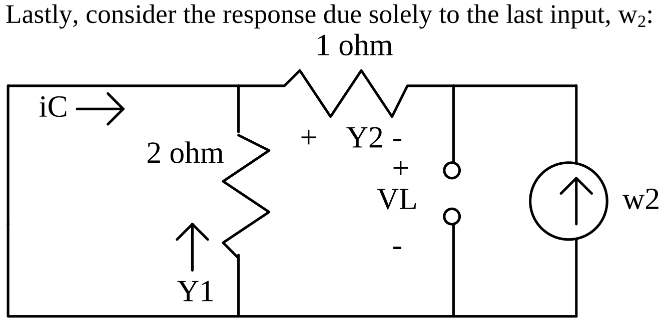
From this simple circuit, we readily find $V_L|_{Vc} = -1V_c$, $i_C|_{Vc} =-0.5V_c$, $Y_1|_{Vc} = 0.5V_c$, and $Y_2|_{Vc} = 0V_c$.

Next, consider the response due solely to the first input, w_1:



From this single source circuit, we find $V_L|_{w1} = 1w_1$, $i_C|_{w1} =0.5w_1$, $Y_1|_{w1} = -0.5w_1$, and $Y_2|_{w1} = 0w_1$.

Lastly, consider the response due solely to the last input, w_2:



Clearly, $V_L|_{w2} = 1w_2$, $i_C|_{w2} = -1w_2$, $Y_1|_{w2} = 0w_2$, and $Y_2|_{w2} = -1w_2$.
Subsituting these values into our state and output equations produces:

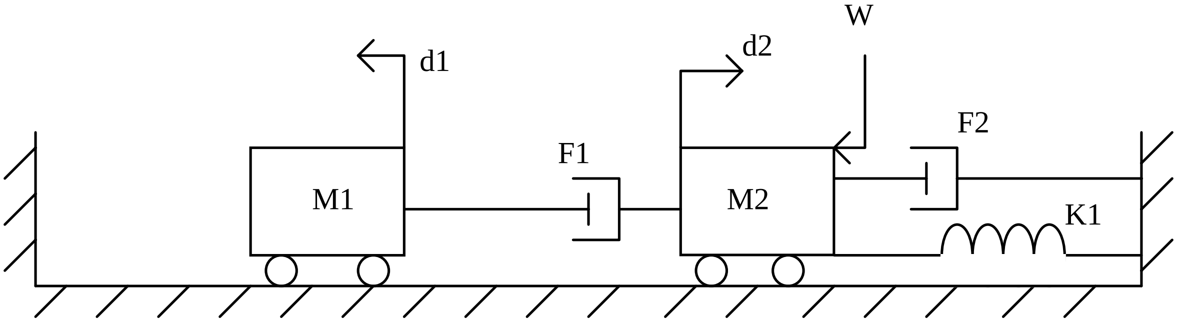
$$\dot{x} = \begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} 2(-i_L + -1v_c + 1w_1 + 1w_2) \\ 3(1i_L + -0.5v_c + 0.5w_1 + -1w_2) \end{bmatrix} \text{ and } y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} (0i_L + 0.5v_c + -0.5w_1 + 0w_2) \\ (1i_L + 0v_c + 0w_1 + -1w_2) \end{bmatrix}$$

Or, expressing these in nice matrix-vector form, we find that:

$$\dot{x} = \begin{bmatrix} -2 & -2 \\ 3 & -3/2 \end{bmatrix} x + \begin{bmatrix} 2 & 2 \\ 3/2 & -3 \end{bmatrix} w, \; x(0) = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

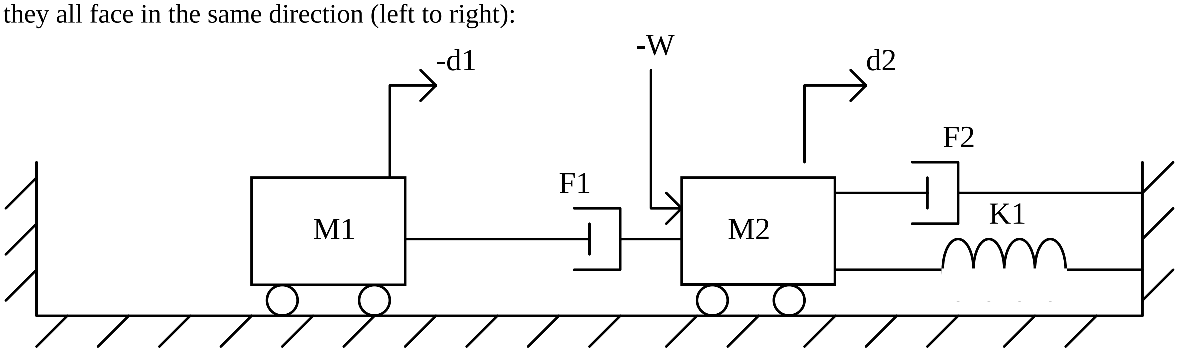
$$y = \begin{bmatrix} 0 & 0.5 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} -0.5 & 0 \\ 0 & -1 \end{bmatrix} w$$

b) Using the current-force analogy you learned today in class, find the analogous electrical circuit for the mechanical system below then write the state variable model.

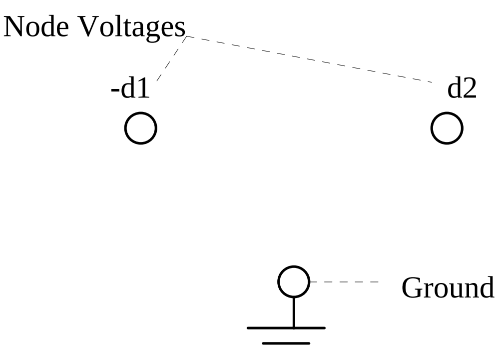


Solution:

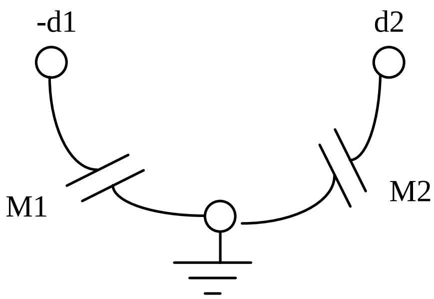
Using the procedure learned in class, first, let's change the direction of the displacements and input force so that they all face in the same direction (left to right):



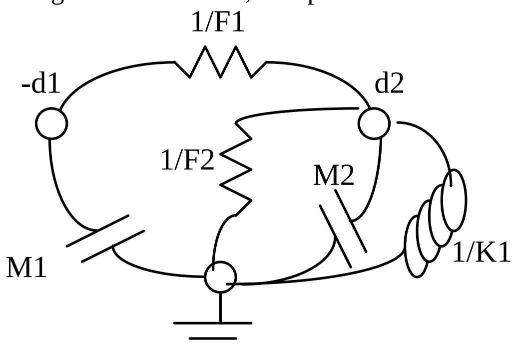
we have a node for each mass plus a reference node with node voltages as the displacements: :



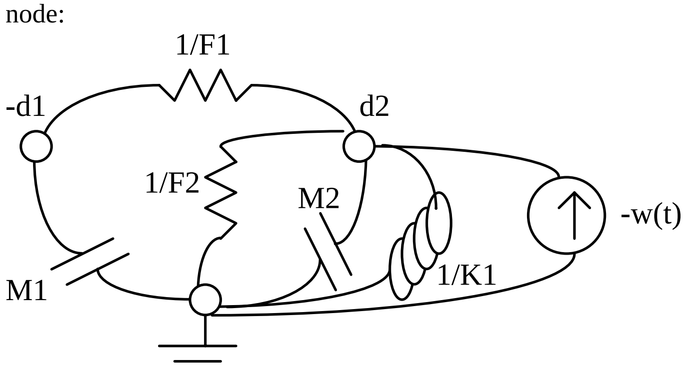
Each mass in the system turns into a capacitor connected from the corresponding "displacement" node to ground:



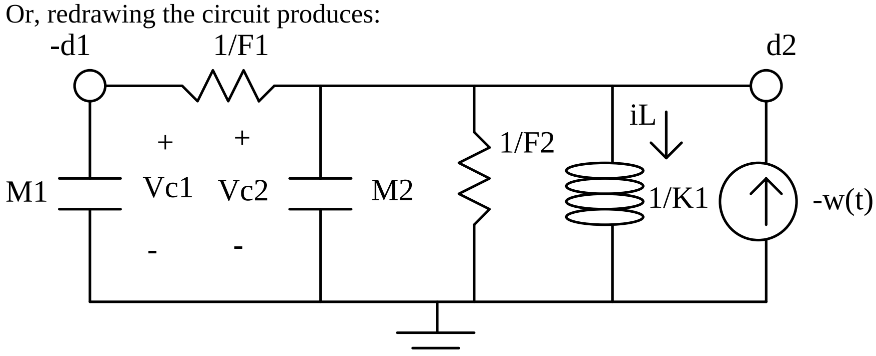
Next, the dashpots become resistors with values of 1/F in ohms and springs turn into inductors with values of 1/K henries. These resistors and inductors are attached to the analogous electrical circuit just as the corresponding dashpot or spring is connected in the mechanical circuit. For example, dashpot F1 becomes a resistor 1/F1 with connected between node (-d1) and node (d2). Spring K1 becomes an inductor 1/K1 connected between node (d2) and ground. Likewise, dashpot F2 becomes a resistor 1/F2 and is also connected from node (d2) to ground.



As a final step, all external input forces become current sources in the analogous electrical circuit, connected in parallel with the corresponding capacitor (mass) and with the direction of current flow going *into* the displacement node:



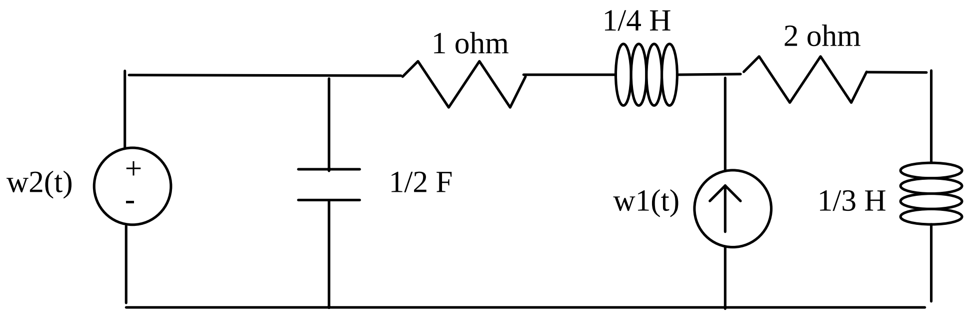
Or, redrawing the circuit produces:



The state variable model for this analogous electrical circuit is very easy to find. If we let $x = \begin{bmatrix} V_{c1} \\ V_{c2} \\ i_L \end{bmatrix} = \begin{bmatrix} -d_1 \\ d_2 \\ K1(d_2) \end{bmatrix}$

then the corresponding state variable model is: $\dot{x} = \begin{bmatrix} -F1/M1 & F1/M1 & 0 \\ F1/M2 & -(F1+F2)/M2 & -1/M2 \\ 0 & K1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/M2 \\ 0 \end{bmatrix} w$

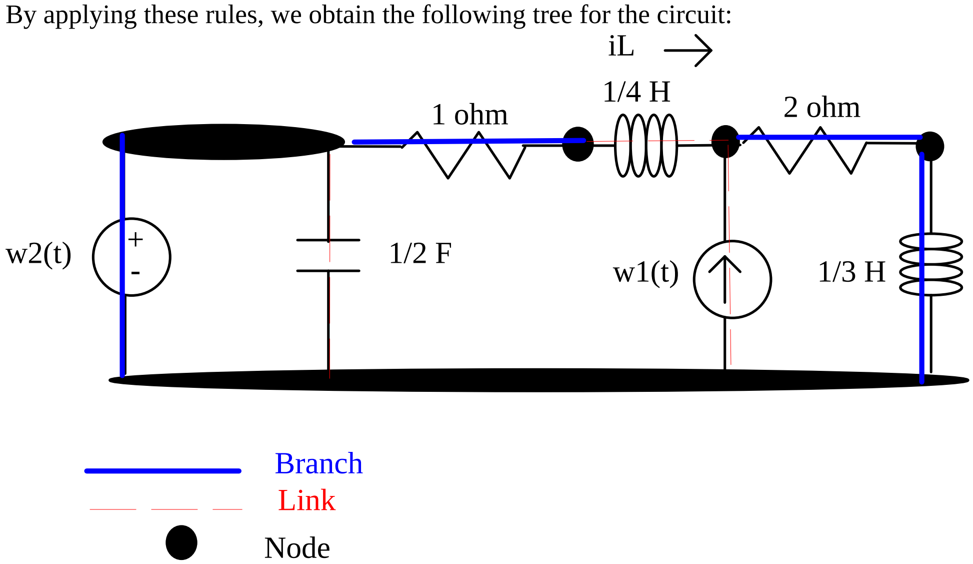
2. a) Find a state variable model for the electrical network shown below. Be careful, there may be redundant state variables!



Solution: to determine the presence of redundant state variables, we should first construct a tree for the circuit with the following convention:

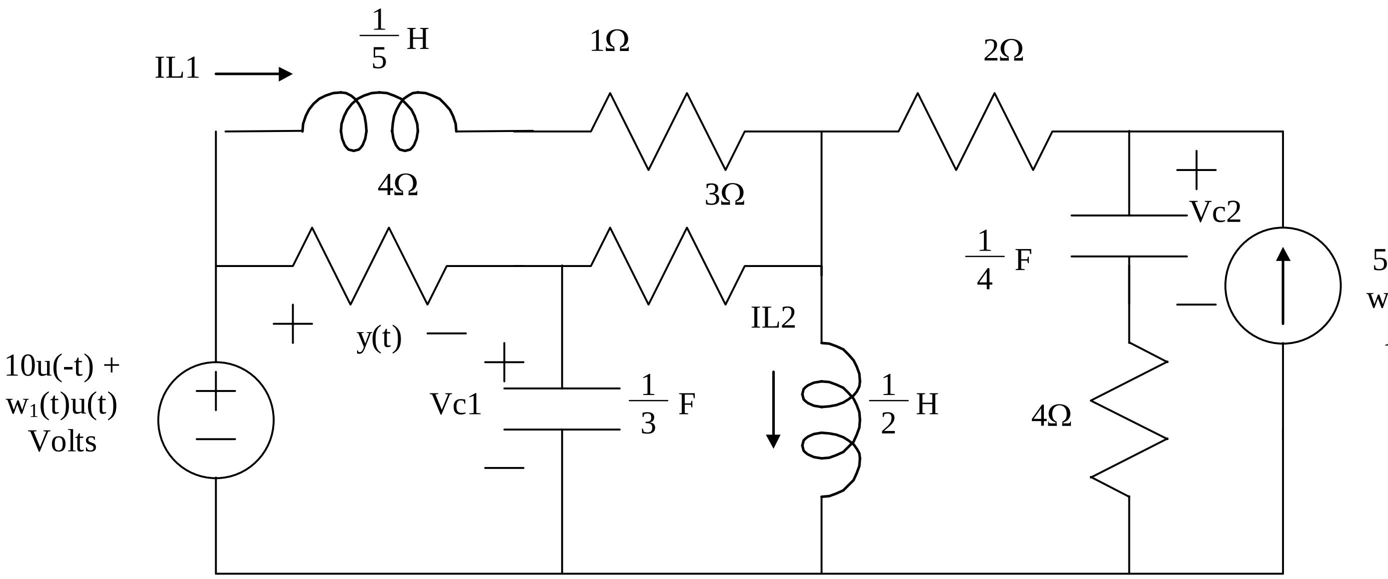
1. Put all independent current sources in the links and all voltage sources in the branches of the tree.
2. Try to put all inductors in the links and capacitors in the branches. If there are some inductors or capacitors for which this is impossible, then the corresponding state variable is redundant (i.e., not independent) and should be eliminated from your minimal set.

By applying these rules, we obtain the following tree for the circuit:



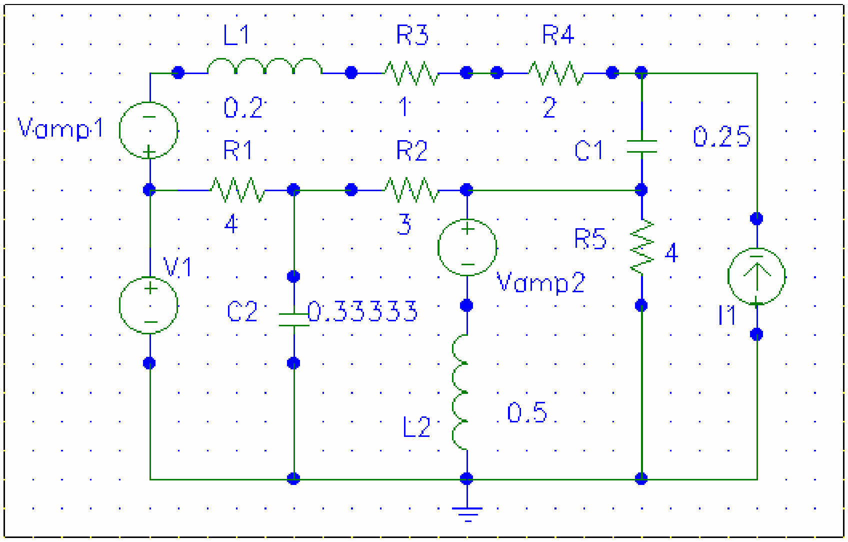
Notice that in order to avoid a close path of branches, we must put the ½ F capacitor into the red links. Conversely, in order for the network of branches to be contiguous and connect to every node of the circuit, we must put either the 1/3 H or ¼ H inductor into the branches (I have chosen to put the 1/3 H into the blue branches). This implies that there is only ONE independent state variable in this circuit: the current in the 1/3 H inductor! Writing the state equation of the form $\dot{i}_L = \frac{1}{L} v_L = 4(v_L)$, we find that $v_L = -3i_L - \frac{1}{3}\dot{i}_L - 2w_1 - \frac{1}{3}\dot{w}_1 + w_2$ or $\dot{i}_L = -12i_L - \frac{4}{3}\dot{i}_L - 8w_1 - \frac{4}{3}\dot{w}_1 + 4w_2$. Grouping all the $\dot{i}_L$ terms on the Left Hand Side, we find the final form of the state variable model:

$\dot{i}_L = -36/7 i_L - 24/7 w_1 - \frac{4}{7}\dot{w}_1 + 12/7 w_2$ or $\dot{x} = \left[\frac{-36}{7}\right]x + \left[\frac{-24}{7} \quad \frac{-4}{7} \quad \frac{12}{7} \quad 0\right]\begin{bmatrix} w_1 \\ \dot{w}_1 \\ w_2 \\ \dot{w}_2 \end{bmatrix}$. Notice that the redundancy causes the derivatives of the inputs to appear in the state variable equation.



Let the states be v_{C1}, v_{C2}, i_{L1}, i_{L2}: $x = [v_{C1} \quad v_{C2} \quad i_{L1} \quad i_{L2}]^T$

Here is the Pspice Schematic for t < 0 (note the dummy voltage sources in series with the inductors to measure current):




```
**** 09/30/98 15:27:44 ******** NT Evaluation PSpice (October 1996) **********
****     CIRCUIT DESCRIPTION


******************************************************************************
* Schematics Version 7.1 - October 1996
* Wed Sep 30 15:27:44 1998
** Analysis setup **
.OP
```

```
* From [SCHEMATICS NETLIST] section of msim.ini:
.lib nom.lib

.INC "hw2.net"

**** INCLUDING hw2.net ****
* Schematics Netlist *

R_R4         $N_0002 $N_0001  2
R_R2         $N_0004 $N_0003  3
R_R1         $N_0005 $N_0004  4
C_C1         $N_0003 $N_0001  0.25
V_V4         $N_0005 $N_0006 DC 0
V_V3         $N_0003 $N_0007 DC 0
I_I1         0 $N_0001 DC 5
C_C2         0 $N_0004  0.33333
V_V1         $N_0005 0 DC 10
R_R5         0 $N_0003  4
L_L1         $N_0006 $N_0008  0.2
R_R3         $N_0008 $N_0002  1
L_L2         0 $N_0007  0.5

**** RESUMING hw2.cir ****
.INC "hw2.als"

**** INCLUDING hw2.als ****
* Schematics Aliases *

.ALIASES
R_R4            R4(1=$N_0002 2=$N_0001 )
R_R2            R2(1=$N_0004 2=$N_0003 )
R_R1            R1(1=$N_0005 2=$N_0004 )
C_C1            C1(1=$N_0003 2=$N_0001 )
V_V4            V4(+=$N_0005 -=$N_0006 )
V_V3            V3(+=$N_0003 -=$N_0007 )
I_I1            I1(+=0 -=$N_0001 )
C_C2            C2(1=0 2=$N_0004 )
V_V1            V1(+=$N_0005 -=0 )
R_R5            R5(1=0 2=$N_0003 )
L_L1            L1(1=$N_0006 2=$N_0008 )
R_R3            R3(1=$N_0008 2=$N_0002 )
L_L2            L2(1=0 2=$N_0007 )
.ENDALIASES


**** RESUMING hw2.cir ****


.probe


.END

****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
******************************************************************************

 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


($N_0001)   25.0000                   ($N_0002)   15.0000

($N_0003)    0.0000                   ($N_0004)    4.2857

($N_0005)   10.0000                   ($N_0006)   10.0000

($N_0007)    0.0000                   ($N_0008)   10.0000




    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    V_V4        -5.000E+00
```

```
V_V3          1.429E+00
V_V1          3.571E+00

TOTAL POWER DISSIPATION   -3.57E+01  WATTS
```

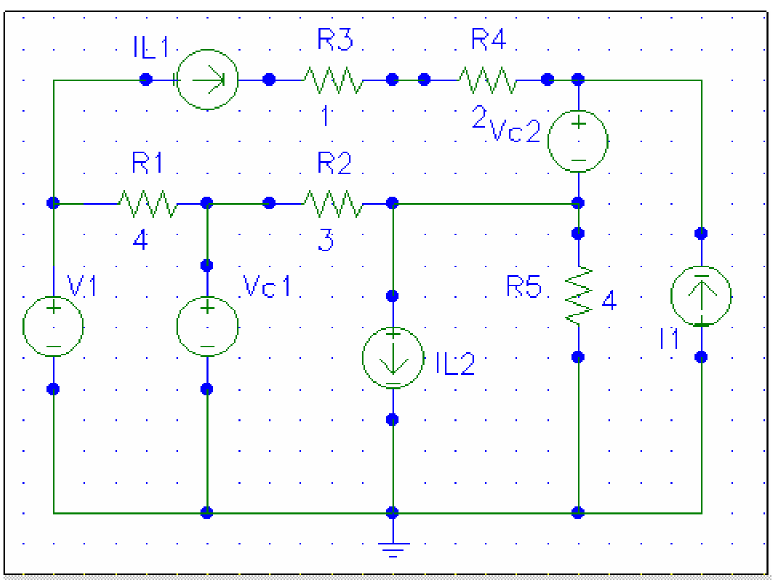
Initial conditions (use steady-state techniques):
$v_{C1}(0)$ = Node Voltage 1 - Node Voltage 3 = 25 V
$v_{C2}(0)$ = Node Voltage 4 = 4.2857 V
$i_{L1}(0)$ = Current thru Vamp1 = -5 A
$i_{L2}(0)$ = Current thru Vamp2 = 1.429 A

Therefore: $x(0) = [25 \quad 4.2857 \quad -5 \quad 1.429]^T$

For t > 0, we have:

$$\dot{x} = \begin{bmatrix} \frac{1}{C_1} i_{C1} \\ \frac{1}{C_2} i_{C2} \\ \frac{1}{L_1} v_{L1} \\ \frac{1}{L_2} v_{L2} \end{bmatrix} = \begin{bmatrix} 3\left(i_{C1}\big|_{v_{C1}} + i_{C1}\big|_{v_{C2}} + i_{C1}\big|_{i_{L1}} + i_{C1}\big|_{i_{L2}} + i_{C1}\big|_{w1} + i_{C1}\big|_{w2} \right) \\ 4\left(i_{C2}\big|_{v_{C1}} + i_{C2}\big|_{v_{C2}} + i_{C2}\big|_{i_{L1}} + i_{C2}\big|_{i_{L2}} + i_{C2}\big|_{w1} + i_{C2}\big|_{w2} \right) \\ 5\left(v_{L1}\big|_{v_{C1}} + v_{L1}\big|_{v_{C2}} + v_{L1}\big|_{i_{L1}} + v_{L1}\big|_{i_{L2}} + v_{L1}\big|_{w1} + v_{L1}\big|_{w2} \right) \\ 2\left(v_{L2}\big|_{v_{C1}} + v_{L2}\big|_{v_{C2}} + v_{L2}\big|_{i_{L1}} + v_{L2}\big|_{i_{L2}} + v_{L2}\big|_{w1} + v_{L2}\big|_{w2} \right) \end{bmatrix}$$

We can use Pspice to fine all of these values by setting each source (i.e., v_{c1}, v_{c2}, i_{L1}, i_{L2}, w_1, and w_2) to 1 in turne while killing the remaining 5 sources. Thus, we replace the capacitors and inductors by voltage sources and current sources and obtain the following Pspice schematic:



By running this model six times (once for each source set to 1 with remaining sources set to 0), we can find all the unknown responses in our state variable. Here is the Pspice output for each of the six cases:

```
****     CIRCUIT DESCRIPTION
******************************************************************************
* Schematics Version 7.1 - October 1996
** Analysis setup **
.OP
* From [SCHEMATICS NETLIST] section of msim.ini:
.lib nom.lib
```

```
.INC "hw2a.net"
**** INCLUDING hw2a.net ****
* Schematics Netlist *

R_R4         $N_0002 $N_0001  2
R_R5         0 $N_0003  4
R_R3         $N_0004 $N_0002  1
R_R2         $N_0005 $N_0003  3
R_R1         $N_0006 $N_0005  4
V_Vc2         $N_0001 $N_0003 DC 0
V_V1         $N_0006 0 DC 0
I_IL1         $N_0006 $N_0004 DC 0
V_Vc1         $N_0005 0 DC 1
I_IL2         $N_0003 0 DC 0
I_I1         0 $N_0001 DC 0

**** RESUMING hw2a.cir ****
.INC "hw2a.als"

**** INCLUDING hw2a.als ****
* Schematics Aliases *

.ALIASES
R_R4            R4(1=$N_0002 2=$N_0001 )
R_R5            R5(1=0 2=$N_0003 )
R_R3            R3(1=$N_0004 2=$N_0002 )
R_R2            R2(1=$N_0005 2=$N_0003 )
R_R1            R1(1=$N_0006 2=$N_0005 )
V_Vc2           Vc2(+=$N_0001 -=$N_0003 )
V_V1            V1(+=$N_0006 -=0 )
I_IL1           IL1(+=$N_0006 -=$N_0004 )
V_Vc1           Vc1(+=$N_0005 -=0 )
I_IL2           IL2(+=$N_0003 -=0 )
I_I1            I1(+=0 -=$N_0001 )
.ENDALIASES

**** RESUMING hw2a.cir ****

.probe


.END
```

```
****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
******************************************************************************

NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
 ($N_0001)     .5714                   ($N_0002)     .5714
 ($N_0003)     .5714                   ($N_0004)     .5714
 ($N_0005)    1.0000                   ($N_0006)    0.0000



    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    V_Vc2       -1.143E-12
    V_V1         2.500E-01
    V_Vc1       -3.929E-01

    TOTAL POWER DISSIPATION   3.93E-01  WATTS
```

$i_{C1}|_{vc1}$ = Current thru V_Vc1 = -0.3929 A
$i_{C2}|_{vc1}$ = Current thru V_Vc2 = 0 A
$v_{L1}|_{vc1}$ = Node Voltage 6 - Node Voltage 4 = -0.5714V
$v_{L2}|_{vc1}$ = Node Voltage 3 - Node Voltage 0 = 0.5714V
$y|_{vc1}$ = Node Voltage 5 - Node Voltage 6 = 1 V

Second run with Vc2 = 1 V and rest of sources killed:

```
 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


($N_0001)    1.0000                   ($N_0002)    1.0000

($N_0003)-3.429E-12                   ($N_0004)    1.0000

($N_0005)    0.0000                   ($N_0006)    0.0000



    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    V_V1         1.000E-12
    V_Vc1       -1.143E-12
    V_Vc2       -2.000E-12

    TOTAL POWER DISSIPATION   2.00E-12  WATTS
```

$i_{C1}|_{vc2}$ = Current thru V_Vc1 = 0 A
$i_{C2}|_{vc2}$ = Current thru V_Vc2 = 0 A
$v_{L1}|_{vc2}$ = Node Voltage 6 - Node Voltage 4 = -1 V
$v_{L2}|_{vc2}$ = Node Voltage 3 - Node Voltage 0 = 0 V
$y|_{vc2}$ = Node Voltage 5 - Node Voltage 6 = 0 V

Third run with IL1 = 1 A and rest of sources killed:

```
 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


($N_0001)    1.7143                   ($N_0002)    3.7143

($N_0003)    1.7143                   ($N_0004)    4.7143

($N_0005)    0.0000                   ($N_0006)    0.0000
```

```
VOLTAGE SOURCE CURRENTS
NAME         CURRENT

V_V1        -1.000E+00
V_Vc1        5.714E-01
V_Vc2        1.000E+00

TOTAL POWER DISSIPATION   0.00E+00  WATTS
```

$i_{C1}|_{iL1}$ = Current thru V_Vc1 = 0.5714 A
$i_{C2}|_{iL1}$ = Current thru V_Vc2 = 1 A
$v_{L1}|_{iL1}$ = Node Voltage 6 - Node Voltage 4 = -4.7143 V
$v_{L2}|_{iL1}$ = Node Voltage 3 - Node Voltage 0 = 1.7143 V
$y|_{iL1}$ = Node Voltage 5 - Node Voltage 6 = 0 V

Fourth run with IL2 = 1 A and rest of sources killed:

```
NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


($N_0001)   -1.7143                   ($N_0002)   -1.7143

($N_0003)   -1.7143                   ($N_0004)   -1.7143

($N_0005)    0.0000                   ($N_0006)    0.0000
```

```
VOLTAGE SOURCE CURRENTS
NAME         CURRENT

V_V1        -1.714E-12
V_Vc1       -5.714E-01
V_Vc2        3.429E-12

TOTAL POWER DISSIPATION   0.00E+00  WATTS
```

$i_{C1}|_{iL2}$ = Current thru V_Vc1 = -0.5714 A
$i_{C2}|_{iL2}$ = Current thru V_Vc2 = 0 A
$v_{L1}|_{iL2}$ = Node Voltage 6 - Node Voltage 4 = 1.7143 V
$v_{L2}|_{iL2}$ = Node Voltage 3 - Node Voltage 0 = -1.7143 V
$y|_{iL2}$ = Node Voltage 5 - Node Voltage 6 = 0 V

Fifth run with w1 = 1 V and rest of sources killed:

```
 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


($N_0001) 1.714E-12                   ($N_0002) 3.714E-12

($N_0003) 1.714E-12                   ($N_0004) 4.714E-12

($N_0005)    0.0000                   ($N_0006)    1.0000
```

```
VOLTAGE SOURCE CURRENTS
NAME         CURRENT

V_Vc1         2.500E-01
V_Vc2         1.000E-12
V_V1         -2.500E-01

TOTAL POWER DISSIPATION   2.50E-01  WATTS
```

$i_{C1}|_{w1}$ = Current thru V_Vc1 = 0.25 A
$i_{C2}|_{w1}$ = Current thru V_Vc2 = 0 A
$v_{L1}|_{w1}$ = Node Voltage 6 - Node Voltage 4 = 1 V
$v_{L2}|_{w1}$ = Node Voltage 3 - Node Voltage 0 = 0 V
$y|_{w1}$ = Node Voltage 5 - Node Voltage 6 = -1 V

Sixth and final run with w2 = 1 V and rest of sources killed:

```
NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE

($N_0001)    1.7143                   ($N_0002)    1.7143
($N_0003)    1.7143                   ($N_0004)    1.7143
($N_0005)    0.0000                   ($N_0006)    0.0000
```

```
VOLTAGE SOURCE CURRENTS
NAME         CURRENT

V_Vc1        5.714E-01
V_Vc2        1.000E+00
V_V1         1.714E-12

TOTAL POWER DISSIPATION   0.00E+00  WATTS
```

$i_{C1}|_{w2}$ = Current thru V_Vc1 = 0.5714 A
$i_{C2}|_{w2}$ = Current thru V_Vc2 = 1 A
$v_{L1}|_{w2}$ = Node Voltage 6 - Node Voltage 4 = -1.7143 V
$v_{L2}|_{w2}$ = Node Voltage 3 - Node Voltage 0 = 1.7143 V
$y|_{w2}$ = Node Voltage 5 - Node Voltage 6 = 0 V

The state equation is

$$\dot{x} = \begin{bmatrix} 3\left(i_{C1}|_{v_{C1}}\ i_{C1}|_{v_{C2}}\ i_{C1}|_{i_{L1}}\ i_{C1}|_{i_{L2}}\right) \\ 4\left(i_{C2}|_{v_{C1}}\ i_{C2}|_{v_{C2}}\ i_{C2}|_{i_{L1}}\ i_{C2}|_{i_{L2}}\right) \\ 5\left(v_{L1}|_{v_{C1}}\ v_{L1}|_{v_{C2}}\ v_{L1}|_{i_{L1}}\ v_{L1}|_{i_{L2}}\right) \\ 2\left(v_{L2}|_{v_{C1}}\ v_{L2}|_{v_{C2}}\ v_{L2}|_{i_{L1}}\ v_{L2}|_{i_{L2}}\right) \end{bmatrix} \underline{x} + \begin{bmatrix} 3\left(i_{C1}|_{w1}\ i_{C1}|_{w2}\right) \\ 4\left(i_{C2}|_{w1}\ i_{C2}|_{w2}\right) \\ 5\left(v_{L1}|_{w1}\ v_{L1}|_{w2}\right) \\ 2\left(v_{L2}|_{w1}\ v_{L2}|_{w2}\right) \end{bmatrix} \underline{w}$$

$$= \begin{bmatrix} 3(-0.3929\ 0\ 0.5714\ \ 0.5714) \\ 4(0\ 0\ 1\ 0) \\ 5(-0.5714\ -1\ -4.7143\ -1.7143) \\ 2(0.5714\ 0\ 1.7143\ 1.7143) \end{bmatrix} \underline{x} + \begin{bmatrix} 3(0.25\ 0.5714) \\ 4(0\ 1) \\ 5(1\ -1.7143) \\ 2(0\ 1.7143) \end{bmatrix} \underline{w}$$

$$= \begin{bmatrix} -1.1787 & 0 & 1.7142 & 1.7142 \\ 0 & 0 & 4 & 0 \\ -2.857 & -5 & -23.5715 & -8.5715 \\ 0 & 1.1428 & 3.4286 & 3.4286 \end{bmatrix} \underline{x} + \begin{bmatrix} 0.75 & 1.7142 \\ 0 & 4 \\ 5 & -8.5715 \\ 0 & 3.4286 \end{bmatrix} \underline{w}$$

The output equation is : $y = [-v_{C1} + w_1] = [-1 \quad 0 \quad 0 \quad 0]\underline{x} + [1 \quad 0]\underline{w}$