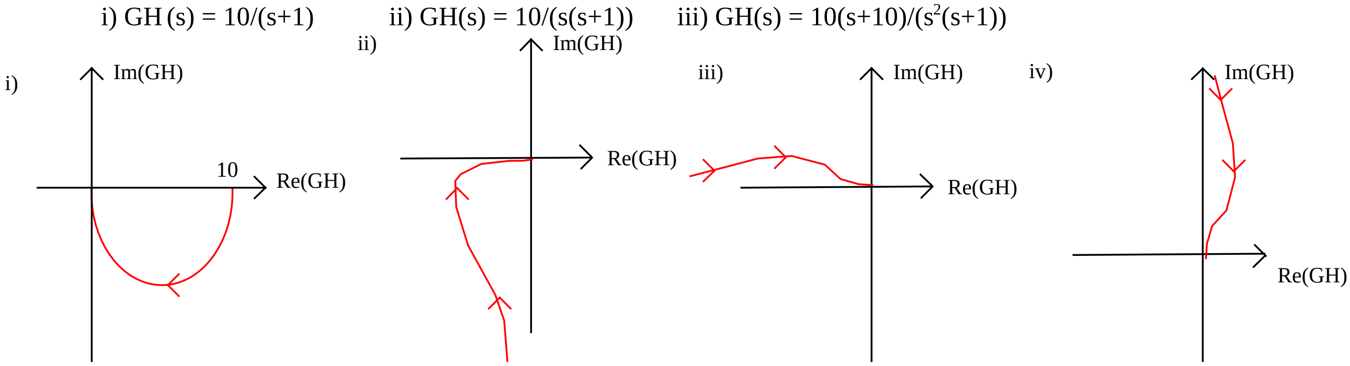
EE571 - Solution to HW#29

1. b)This is a type 0 system. From the plot, the steady-state value is 8 = 1xK/p. Also, the time constant is about 0.25 seconds. Thus, 1/p=0.25 or p=4 and K=32. Thus, the first order transfer function is 32/(s+4)

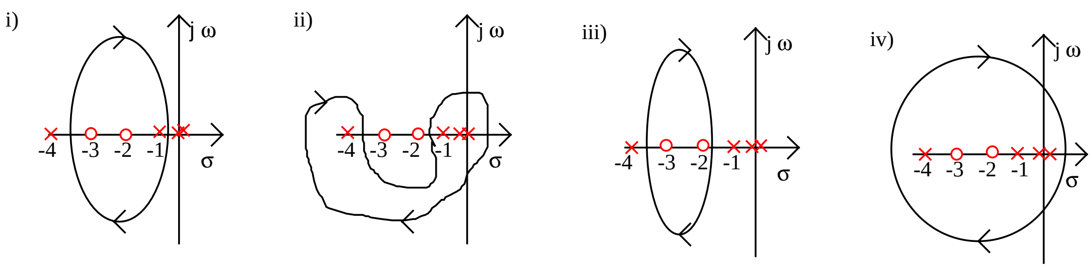
c) Make a polar plot of:

i) GH (s) = 10/(s+1) ii) GH(s) = 10/(s(s+1)) iii) GH(s) = 10(s+10)/(s^2(s+1))



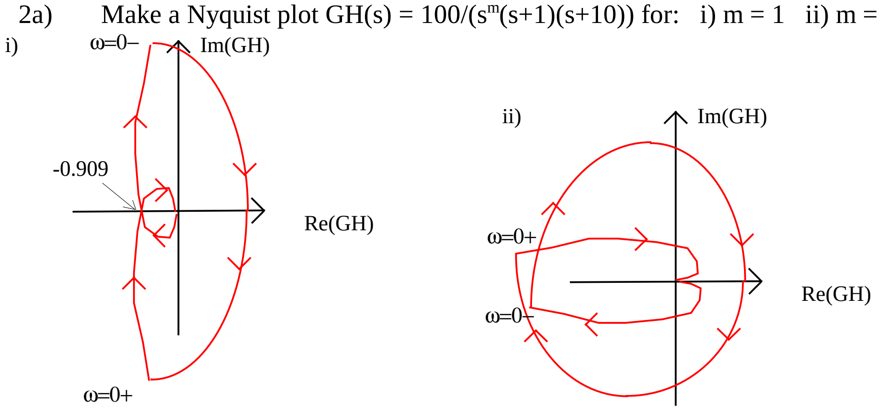
iv) GH(s) = 10(s+10)/(s^3(s+1))

d) Suppose that GH(s) = 10(s+2)(s+3)/(s^2(s+1)(s+4). Use the mapping theorem to determine the number of closckwise encirclements of the origin the following contours would make if we mapped them using GH(s) (Don't map! just find N)



N=Z-P=2-1=1 N=Z-P=0-4=-4 or 4 ccw N=Z-P=2-0=2 N=Z-P=2-4=-2 or 2 ccw

2a) Make a Nyquist plot GH(s) = 100/(s^m(s+1)(s+10)) for: i) m = 1 ii) m = 2



b) Find the gain and phase margins for problem 2a). Are the closed-loop systems stable?

i) $\mathrm{Im}(GH(j\omega_{cp})) = 0 = \mathrm{Im}\left(\frac{100}{j\omega_{cp}(j\omega_{cp}+1)(j\omega_{cp}+10)}\right) \Rightarrow \omega_{cp} = \sqrt{10} = 3.1623 \Rightarrow gm = \frac{1}{|GH(j\omega_{cp})|} = 1.1 = 0.828\,dB$

$|GH(j\omega_{cg})|^2 = 1^2 = \left(\frac{100^2}{\omega_{cg}^2(\omega_{cg}^2+1^2)(\omega_{cg}^2+10^2)}\right) \Rightarrow \omega_{cg} = 3.0145 \Rightarrow pm = 180° + \angle GH(j\omega_{cg}) = 1.5763°$ The closed-loop system is just barely stable!!!

ii) $\mathrm{Im}(GH(j\omega_{cp})) = 0 = \mathrm{Im}\left(\dfrac{100}{-\omega_{cp}^{2}(j\omega_{cp}+1)(j\omega_{cp}+10)}\right) \Rightarrow \omega_{cp} = 0 \Rightarrow gm = \dfrac{1}{|GH(j\omega_{cp})|} = 0 = -\infty\, dB$

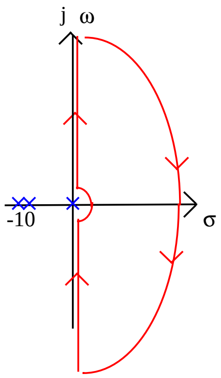
$|GH(j\omega_{cg})|^2 = 1^2 = \left(\dfrac{100^2}{\omega_{cg}^{4}(\omega_{cg}^{2}+1^2)(\omega_{cg}^{2}+10^2)}\right) \Rightarrow \omega_{cg} = 2.0657 \Rightarrow pm = 180° + \angle GH(j\omega_{cg}) = -75.84°$ The closed-loop system is very unstable!!!

c) Given the open-loop transfer function, $GH(s) = K/(s(s+10)^2)$, use Nyquist stability to determine the range on K for which the closed-loop system is unstable (be sure to consider K < 0)

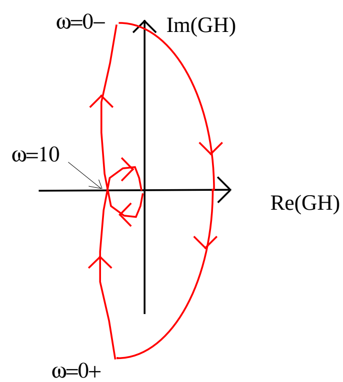
For K > 0, $\mathrm{Im}(GH(j\omega_{cp})) = 0 = \mathrm{Im}\left(\dfrac{K}{j\omega_{cp}(j\omega_{cp}+10)^2}\right) \Rightarrow \omega_{cp} = 10\, r/s \Rightarrow |G(j\omega_{cp})| = K/2000$. $\therefore$ the system is stable for K <2000.

For the case of K <0, to obtain the Nyquist Plot, we just need to rotate the Nyquist Plot for K > 0 by 180 degrees. As be seen from the resulting Nyquist plot, N=Z=1 and thus the system has 1 RHP closed-loop pole for all K < 0. Hence, the range on K for which the closed-loop system is stable is 0 < K < 2000

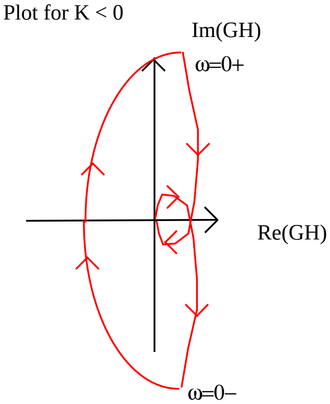
Nyquist Path



Nyquist Plot for K > 0



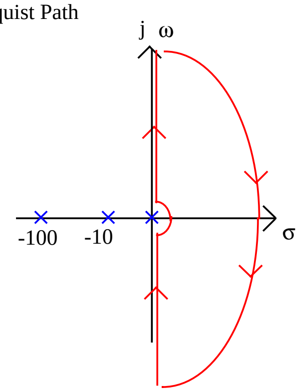
Nyquist Plot for K < 0



d) i) $G(s)=K/(s^m(s+1)(s/100+1))$. The slope at low frequencies is -20 dB, hence m=1 and K=1000. Thus, $G(s)=1000/(s^1(s+1)(s/100+1))$. ii) $e_{ss}|_{step}=1/(1+K_p)=0$ iii) $e_{ss}|_{ramp}=1/K_v=1/1000$. iv) From Bode Plot $\omega_{cg} \cong 31\, r/s$ v) From Bode Plot $\omega_{cp} \cong 10\, r/s$. v) From Bode Plot gm=-20 dB and pm= -15 deg., hence the closed-loop system is UNSTABLE!

e) The simplest compensator is $G_c=K_c=1/100$.

Nyquist Path



Nyquist Plot for K > 0