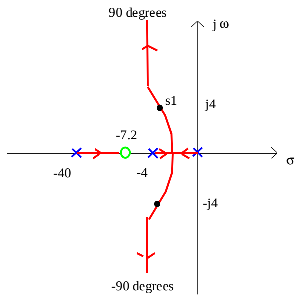
Solution to HW#22

1.a) Design a PD (ultimate lead compensator) to meet the transient specs given in problem 2 on HW21.

Sol'n: Desired dominant poles are at s=-4+j4 and s=-4-j4. The angle of deficiency is still the same as it was for the lead design: $\angle G_{PD}(s_1) = 180^\circ \times odd\# - \angle G(s_1) = 180^\circ \times odd\# - \angle \frac{20}{s_1(s_1+4)(s_1+40)} = 180^\circ - 128.66^\circ = 51.34^\circ$. Therefore, our PD compensator must supply 51.34 degrees. The form of G_{PD} is $G_{PD}(s)=K(s+z)$. Then $\angle G_{PD}(s_1) = 51.34^\circ = \angle(s_1 + z)$. Solving for z_c: $IM(s_1)/RE(s_1+z) = \tan(51.34^\circ) or\ z = IM(s_1)/\tan(51.34^\circ) - RE(s_1) = 7.2$. The last step is to find K from the magnitude condition: $K = 1/|G(s_1)(s_1+7.2)| = 8$. Thus, the PD compensator is $G_{PD}(s)=8(s+7.2)=8s+57.6$.

b) Sketch the resulting compensated root locus of $G(s)G_{PD}(s)$

Sol'n:



Lead Root Locus

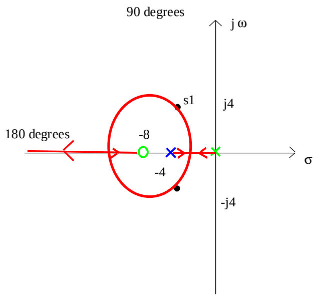
c) Design a PI (ultimate lag compensator) compensator for $G(s) = \dfrac{10}{s+4}$ such that the following specs are met:

1. t_s = 1 second
2. Damping coefficient of ζ = 0.707
3. $ess|_{step} = 0$

Sol'n: Notice we need a type 1 system to meet the e_{ss} specs. If we lump the needed integrator in with G(s) then our new open-loop transfer function is G(s)/s = $\dfrac{10}{s(s+4)}$ Desired dominant poles are again at s=-4+j4 and s=-4-j4. By lumping the integrator in with G(s), we can now can do a PD compensator design on the new open-loop transfer function, G(s)/s. The angle of deficiency is $\angle G_{PD}(s_1) = 180^\circ \times odd\# - \angle G(s_1)/s_1 = 180^\circ \times odd\# - \angle \frac{10}{s_1(s_1+4)} = 180^\circ - 225^\circ = 45^\circ$. Therefore, our PD compensator must supply 45 degrees. The form of G_{PD} is $G_{PD}(s)=K(s+z)$. Thus, $\angle G_{PD}(s_1) = 45^\circ = \angle(s_1+z)$. Solving for z_c: $IM(s_1)/RE(s_1+z) = \tan(45^\circ) or\ z = IM(s_1)/\tan(51.34^\circ) - RE(s_1) = 8$. The last step is to find K from the magnitude condition: $K = 1/|G(s_1)(s_1+7.2)/s_1| = 0.4$. Thus, the PI compensator is $G_{PI}(s)=0.4(s+8)/s$ = 0.4+3.2/s.

d) Sketch the resulting compensated root locus of $G(s)G_{PI}(s)$

Sol'n:



e) Design a PID (ultimate lead-lag compensator) compensator for $G(s) = \frac{10}{s+4}$ such that the following specs are met:

1. ts = 0.5 second
2. Damping coefficient of = 0.707
3. ess|ramp = 0.1

Sol'n:

A PID compensator is of the form of $G_{PID}(s) = K_D s + K_P + K_I/s$. The PID design process is unique among our root locus-based compensator designs. The first step, is to satisfy the steady-state error specs. Note that the desired K_V is 10= K_i x 10/4 which implies that K_i = 4. Now, $ess|_{ramp}$ = 0.1 as specified. The next step is to find K_P and K_D from:

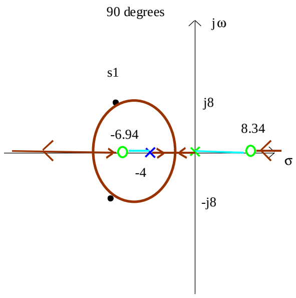
$$K_P + K_D s_1 = -1/G(s_1) - K_I/(s_1).$$

Here, once we pick a pair of desired dominant poles, then K_P and K_D are uniquely determined. By virtue of the transient specifications, we have no option in finding s_1. In fact, s_1 = -8+j8 is the only possible location to produce a damping ratio of 0.707 and a settling time of 0.5. Thus, $K_P + K_D s_1 = K_P - 8K_D + j8K_D = 0.65 - j0.55$. By equating the imaginary part of this equation we find that K_D = 0.55/8 = - 0.0688. By substituting this value and equating the real part of our equation, we find that K_P = 0.65-K_D x Re(s_1) = 0.1 Thus, $G_{PID}(s) = K_D s + K_P + K_I/s$ = -0.0688s+0.1+4/s. According to MATLAB, the closed-loop poles for the PID compensated system are -8+j8 and -8-j8. The zeroes of the closed-loop system are located at 8.34 and –6.94.

f) Sketch the resulting compensated root locus of GPID(s)G(s)

Sol'n:

Note that there are two disturbing facts about this design. The overall K=K_D is NEGATIVE and there is an open-loop (and closed-loop) zero in the RHP. Thus, to perform the root locus, we actually have to use the "complementary" root locus plot which is based upon K being negative: Here is a plot of both the root-locus and the complementary root locus for this problem. The regular root locus is shown in cyan (light blue) and the complementary root locus is shown in brown:



Don't worry about the complementary root locus!

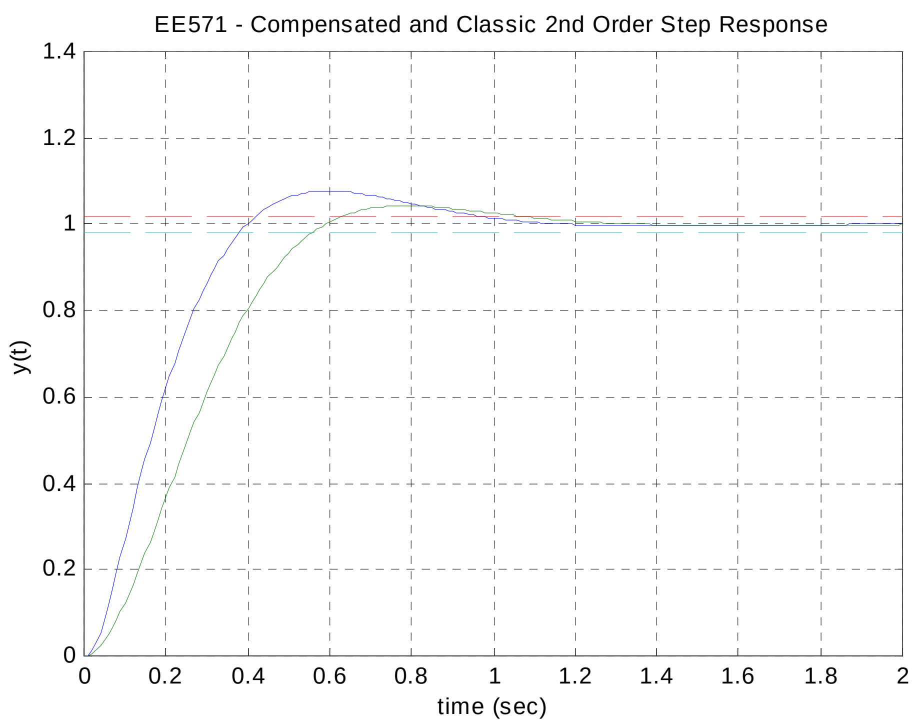
2. For my lead design in HW#21, I found the following closed-loop transfer function:

$$\frac{GcG}{1+GcG} = \frac{15872(s+6.9360)}{(s+4-j4)(s+4+j4)(s+33.59)(s+102.41)} = \frac{15872(s+6.9360)}{(s^2+8s+32)(s+33.59)(s+102.41)}$$

If my system were truly a classic second order system with dominant poles at s_1 = –4+j4 and s_2 = -4-j4, the closed-loop transfer function would be:

$$\frac{G_{classic}}{1+G_{classic}} = \frac{32}{s^2+8s+32}$$

I have gone back and plotted my lead compensated step response versus what a classic 2nd order step response would look like for the (see plot below).



a) What are the theoretical values of t_p, M_p , and t_s for dominant poles at s_1 = –4+j4?

Sol'n: $t_p = \frac{\pi}{\omega_d}$ = 0.7854, M_p = 4.32%, and t_s = 1 sec.

b) Given your answer to part d) which plot is which (hint: the classic second-order system should have the correct overshoot and settling time)?

Sol'n: The green plot has the theoretical values of t_p, M_p, and t_s.

c) What are the values of t_p, M_p , and t_s for my lead design from the Matlab step response?

Sol'n: Looking at the blue plot, t_p= 0.6 sec, M_p = 8 %, and t_s = 0.98 sec.

d) Note that the zero at s=-6.936 is fairly close to the dominant poles at $s_1 = -4+j4$. Given the difference between the two plots, what is your conjecture as to the effect of a zero near a pair of dominant poles?

My conjecture is that the presence of a zero near our closed-loop dominant poles causes more overshoot, a faster peak time, and a slightly faster settling time. See the handout on the effect of poles-zeroes for more illustrations of what effect extra poles and zeros might have on the classic step response.