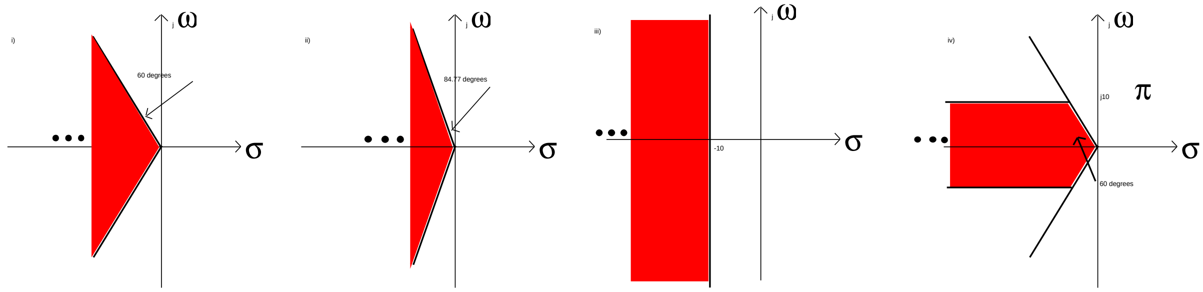
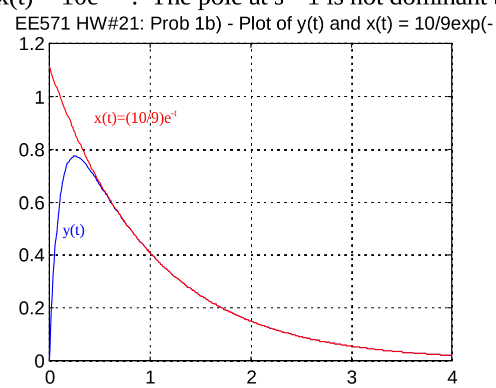
EE571 Solution to HW#21

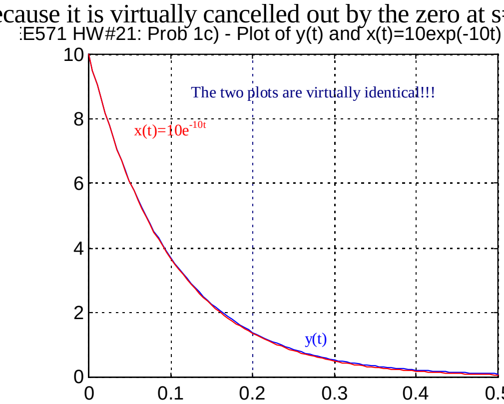
1. a) Find regions in the complex S-plane where dominant 2nd order poles can be placed to satisfy the following transient response specifications:



1b) The pole at s=-1 is dominant. The impulse response is $y(t)=(10/9)e^{-t} - (10/9)e^{-10t}$. See the plot below of the impulse response with the function, $x(t)=(10/9)e^{-t}$.

c) Now, the pole at s=-10 is dominant because the zero at s=-1.05 essentially cancels the pole at s=-1. The impulse response is $y(t)=179/18e^{-10t} +1/18e^{-t}$. The following is a plot of the impulse response and the function $x(t) = 10e^{-10t}$. The pole at s=-1 is not dominant because it is virtually cancelled out by the zero at s=-1.05!!!





A US Navy A-6 intruder attack jet has the following open-loop transfer function for its YAW dynamics:

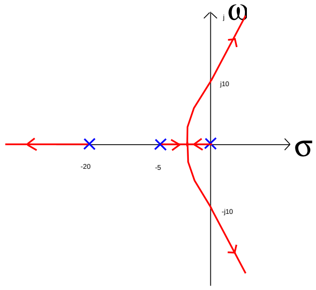
$$G(s) = \frac{30}{s(s+5)(s+20)}, \quad H(s) = 1$$

d) The intruder transfer function is a type 1 system. Thus, $ess|_{unit\ step} = 1/(1+K_p) = 0$ and $ess|_{unit\ ramp} = 1/K_v = 100/30 = 10/3$.

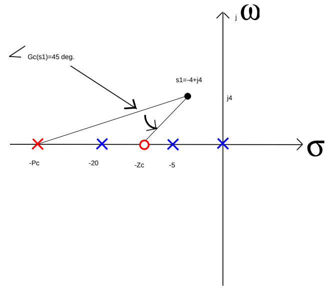
e) (See root-locus below). To determine when the closed-loop system becomes unstable, we need to construct the Routh Array for the characteristic equation, $s^3+100s^2+25s+K=0$. The closed-loop system is unstable for 0 < K < 83.33. The root locus crosses at s=-j10 and s=+j10. Thus, the system will become unstable if we insert a gain greater than 83.33 into the forward loop.

Routh Array

s^3	1	25
s^2	100	30K
s^1	25-0.3K	
s^0	30K	



Uncompensated Root-Locus

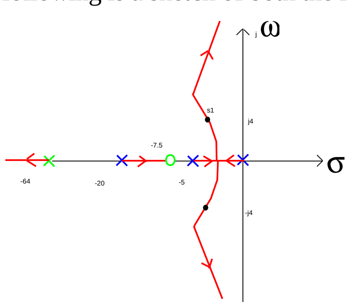


Angle of Deficiency Illustration

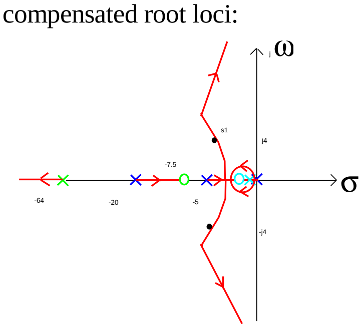
2.a) Desired dominant poles are at s=-4+j4 and s=-4-j4. From the root locus sketch above, it is clear that the uncompensated root locus does **not** pass thru these poles. The angle of deficiency is $\angle G_c(s_1) = 180^\circ \times odd\# - \angle G(s_1) = 180^\circ \times odd\# - \angle \frac{30}{s_1(s_1+5)(s_1+20)} = 180^\circ - 135^\circ = 45^\circ$. Therefore, our lead compensator must supply 45 degrees. The form of G_c is $G_c(s)=K_c(s+z_c)/(s+p_c)$. Let's pick z_c to be as far to the left as possible. Let z_c be 7.5 (by geometry, z_c must be less than 8). Then $\angle G_c(s_1) = 45^\circ = \angle(s_1+7.5) - \angle(s_1+p_c) = 48.81^\circ - \angle(s_1+p_c)\ or\ \angle(s_1+p_c) = 3.81^\circ$. Solving for p_c: $IM(s_1)/RE(s_1+p_c) = \tan(3.81^\circ)\ or\ p_c = IM(s_1)/\tan(3.81^\circ) - RE(s_1) = 64$. The last step is to find K_c from the magnitude condition: $K_c = 1/|G(s_1)(s_1+z_c)/(s_1+p_c)| = 145.07$. Thus, the lead compensator is $G_c(s)=145.07(s+7.5)/(s+64)$.

2.b) To meet the $ess|_{ramp} = 1/20$ spec, we need to add a **lag** compensator of the form $G_{lag} = K_{lag}(s+z_{lag})/(s+p_{lag})$ where K_{lag} is about 1 and $p_{lag} < z_{lag}$. To meet the spec, we must have $ess|_{ramp} = 1/20 = 1/K_v$ or $K_v = 20$. Or, $20 = K_v = \lim_{s\to 0} sG_{lag}G_cG(s) = (z_{lag})(30)(145.07)(7.5)/((p_{lag})(64)(5)(20)) = (z_{lag})(5.1)/(p_{lag})$ Thus, if we choose $p_{lag} = 0.01$, then $z_{lag} = 20p_{lag}/(5.1) = 0.0392$. Finally, we can find K_{lag} from the magnitude condition: $K_c = 1/|G_cG(s_1)(s_1+z_{lag})/(s_1+p_{lag})| = 1.0037$. Thus, the lag compensator is $G_{lag}(s)=1.0037(s+0.0392)/(s+0.01)$.

2.c) The following is a sketch of both the lead compensated and the lead-lag compensated root loci:



Lead Root Locus



Lead/Lag Root Locus

2.d) The open-loop lead compensated transfer function is $G_cG(s)=145.07(s+7.5)(30)/(s(s+5)(s+20)(s+64))$. Thus, the closed-loop poles are the roots of $1+G_cG=0$ which are -4+j4, -4-j4, -15.6, -65.4 (according to MATLAB's roots() function). The open-loop lead/lag compensated transfer function is $G_{lag}G_cG(s)=1.0037(145.07)(s+0.0392)(s+7.5)(30)/(s(s+0.01)(s+5)(s+20)(s+64))$. Thus, the closed-loop poles are the roots of $1+G_{lag}G_cG=0$ which are -0.0394, -3.99+j4.01, -3.99-j4.01, -15.6, -65.4 (according to MATLAB's roots() function). Note, that for the closed-loop Lead/Lag system, the closed-loop pole at s=-0.0394 is NOT the dominant pole because it is effectively canceled by the zero of the LAG compensator located at s=-0.0392. Also, note that the

Lead/Lag root locus does not quite pass through the desired dominant poles at s1=-4+j4 but passes through -3.99+j4.01.