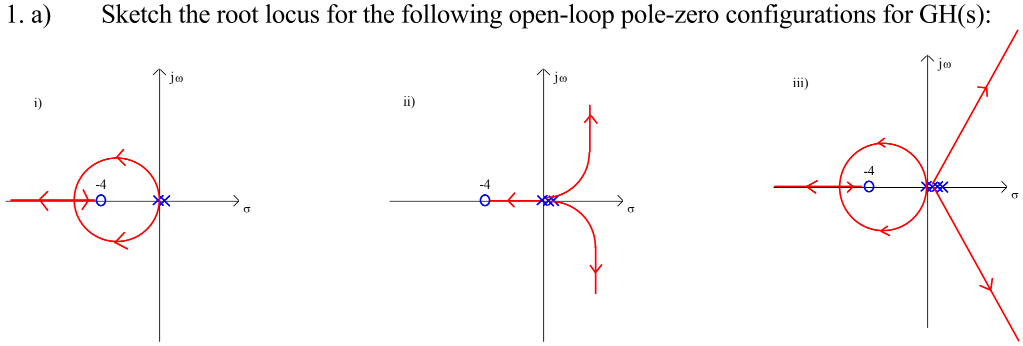
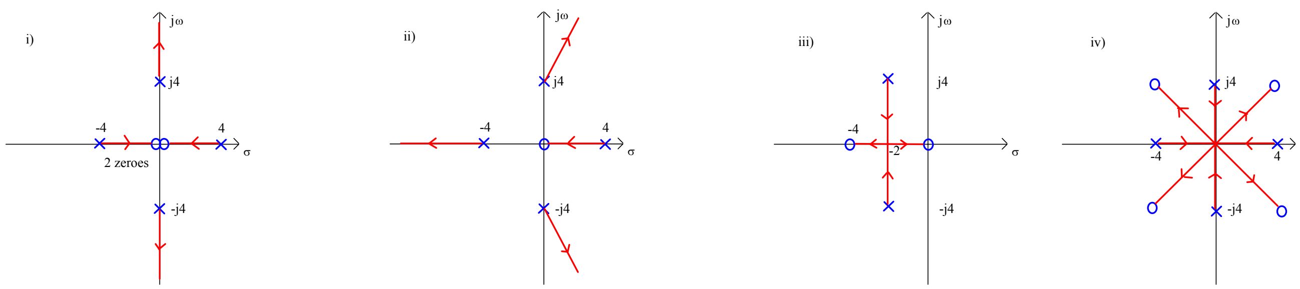
EE571 - HW#20

1\. a) Sketch the root locus for the following open-loop pole-zero configurations for GH(s):

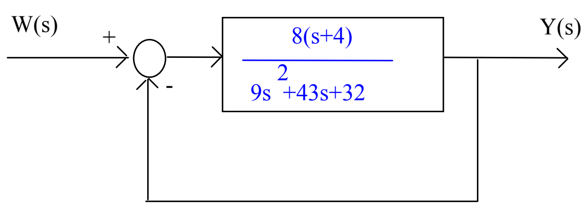


b) Use the property of symmetry (if applicable) to help you sketch the root locus for the following open-loop pole-zero configurations for GH(s):



2\. a)

Solution: Replace G by G/[1+G(H-1)] = 8(s+4)/[s(s+3)+8(s+4)(s+2-1)]



2b) Find the type number, Kp, Kv, Ka, and $ess|_{step}$ $ess|_{ramp}$, and $ess|_{parabola}$ for the following unity feedback systems with

i) G(s)=5(s+2)/[s(s+4)] Solution:

$$K_p = \lim_{s\to 0} G(s) = \infty \therefore ess|_{step} = 1/(1+K_p) = 0$$

$$K_v = \lim_{s\to 0} sG(s) = 10/4 \therefore ess|_{ramp} = 1/K_v = 4/10$$

$$K_a = \lim_{s\to 0} s^2 G(s) = 0 \therefore ess|_{parabola} = 1/K_a = \infty$$

ii) G(s)=5(s+2)/[$(s+4)^2$] Solution:

$$K_p = \lim_{s\to 0} G(s) = 10/16 \therefore ess|_{step} = 1/(1+K_p) = 16/26$$

$$K_v = \lim_{s\to 0} sG(s) = 0 \therefore ess|_{ramp} = 1/K_v = \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s) = 0 \therefore ess|_{parabola} = 1/K_a = \infty$$

iii) G(s)=5(s+2)/[s^2(s+4)] Solution:

$$K_p = \lim_{s\to 0} G(s) = \infty \therefore ess|_{step} = 1/(1+K_p) = 0$$

$$K_v = \lim_{s\to 0} sG(s) = \infty \therefore ess|_{ramp} = 1/K_v = 0$$

$$K_a = \lim_{s\to 0} s^2G(s) = 10/4 \therefore ess|_{parabola} = 1/K_a = 4/10$$

2c) Suppose I used the input w(t)=10u(t)+2r(t) to the unity-feedback systems described in part 2b). What is the new steady-state error (be careful)?

i) G(s)=5(s+2)/[s(s+4)] Hence, $ess = 10ess|_{step} + 2ess|_{ramp} = 0 + 8/10 = 8/10$

ii) G(s)=5(s+2)/[(s+4)^2] Hence, $ess = 10ess|_{step} + 2ess|_{ramp} = 160/26 + \infty = \infty$

iii) G(s)=5(s+2)/[s^2(s+4)] Hence, $ess = 10ess|_{step} + 2ess|_{ramp} = 0 + 0 = 0$

2d) Suppose I used the input w(t)=10u(t)+20u(t-3)+2r(t)+5r(t-8) to the unity-feedback systems described in part 2b). What is the new steady-state error (be careful)?

i) G(s)=5(s+2)/[s(s+4)]

Hence, $ess = 10ess|_{step} + 20ess|_{step} + 2ess|_{ramp} + 5ess|_{ramp} = 0 + 0 + 8/10 + 20/10 = 28/10$

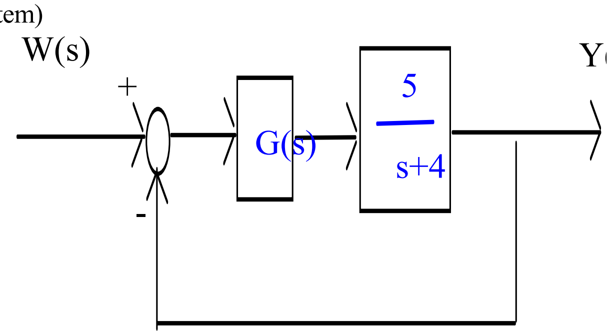
ii) G(s)=5(s+2)/[(s+4)^2]

Hence, $ess = 10ess|_{step} + 20ess|_{step} + 2ess|_{ramp} + 5ess|_{ramp} = 160/26 + 320/26C + \infty + \infty = \infty$

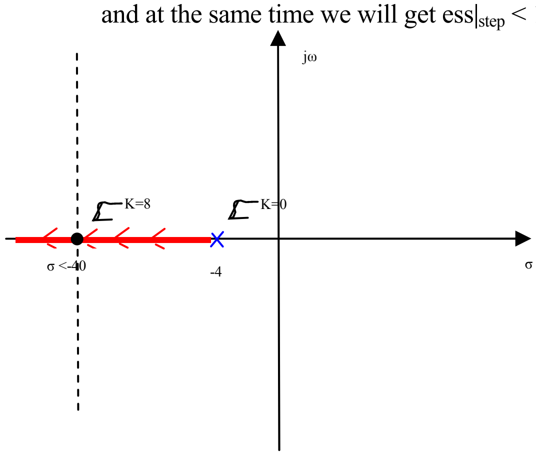
iii) G(s)=5(s+2)/[s^2(s+4)] Hence,

$ess = 10ess|_{step} + 20ess|_{step} + 2ess|_{ramp} + 5ess|_{ramp} = 0 + 0 + 0 + 0 = 0$

2e) For the system shown below, design the simplest compensator possible so that the following system has $ess|_{step} < 1/10$ and a settling time of less than 0.1 seconds and NO OVERSHOOT. (Hint: Try Gc(s)=K and sketch root locus. Find a K that meets both the transient and ess specs. Note that your closed-loop system is a simple 1st order system and not a classic second-order system)

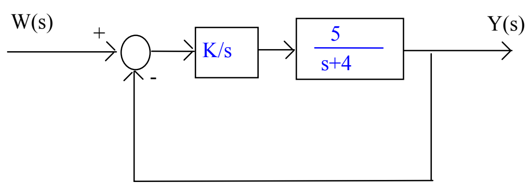


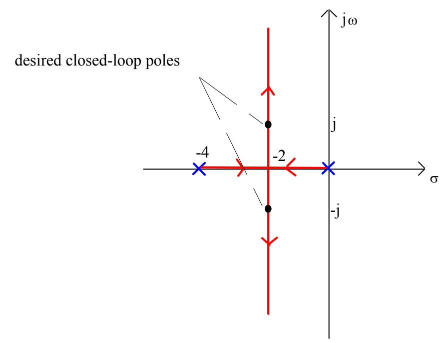
Solution: The simplest sol'n would be to use just a gain for the compensator. To meet the ess spec, we must have Kp > 9 (then 1/(1+Kp) would be < 1/10). From the transient spec, we need a dominant pole on the real axis to the left of the point σ = -40 so that or settling time is less than 0.1 seconds. From the root locus below, we see that the root locus passes thru the desired region. Let's pick a dominant pole at s1 = -50 and use the magnitude condition to find K = 1/|GH(s1)| = 46/5 = 9.2. This value of K produces a Kp of 46/4 = 11.5. Thus, our $ess|_{step}$ = 1/(12.5) which is less than 1/10 and one possible Gc is Gc = K = 9.2. (Note that any K > 8 will work. That is, we will get a dominant pole to the left of σ = -40 and at the same time we will get $ess|_{step}$ < 1/(11).)



2f) Repeat part e) with the specs that $ess|_{step} = 0$ and the closed-loop poles s= -2+j and -2-j. (Hint: Recall that inserting an integrator (1/s) increases the system type number by one. Think about what the minimum system type you need to meet ess specs and lump the 1/s term in with G(s) and sketch the new root locus. If the root locus of G(s)/s passes thru the point s1=-2+j, then use the magnitude condition to find the appropriate K. Gc(s) would then be K/s)

Solution: We need a type 1 system to meet the error specification that $ess|_{step} = 0$. Since we currently have a type 0 system, we must increase the system type by inserting an integrator into the forward loop. Let's look at the new root locus if we do this:





Since the root locus passes through our desired closed-loop poles, all we need do is vary the gain to meet the specifications. We can use the magnitude condition to find K: $K = 1/|GH(s_1)| = |(s(s+4))/5|_{s=-2+j} = 1$

Thus, the simplest compensator possible is Gc(s)=K/s=1/s.

2g) What is the settling time and percent overshoot for the compensated system in 2d)?
Solution:

$$t_s = 4/(\zeta\omega_n) = -4/(-2) = 2\,\text{sec}.$$

$$\zeta = \cos\theta = \cos(\tan^{-1}(\frac{1}{2})) = 0.89 \therefore M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\% = 0.19\%$$