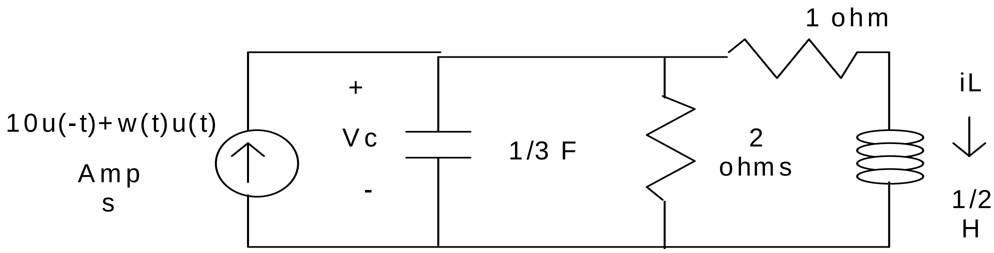
EE571 Solution to HW#1

1. a) Find the state variable model of the form $\dot{x} = Ax + Bw, \quad x(0^+)$ for the following electrical network:



(Hint: Let $x = \begin{bmatrix} v_C \\ i_L \end{bmatrix}$) ((make sure to include your initial conditions))

Solution: Let $x = \begin{bmatrix} v_C \\ i_L \end{bmatrix}$. Note that prior to t=0, the input is 10 amps and the capacitor acts like an open circuit while the inductor behaves as a short circuit. Thus, $i_L(0^-)$ is 6.67 amps and $V_c(0^-)$ is 6.67 volts. For t > 0, the input is w(t) and we can write a KCL and KVL to try to obtain our state equations:

KCL at top node: $-w(t) + 1/3\frac{dV_c}{dt} + \frac{V_c}{2} + i_L = 0$

Therefore, $\frac{dV_c}{dt} = -3/2V_c - 3i_L + 3w(t)$

KVL around loop:

$1/2\frac{di_L}{dt} - V_c + 2i_L = 0$.

Therefore, $\frac{di_L}{dt} = 2V_c - 4i_L$

We can rearrange these two equations into the desired form:

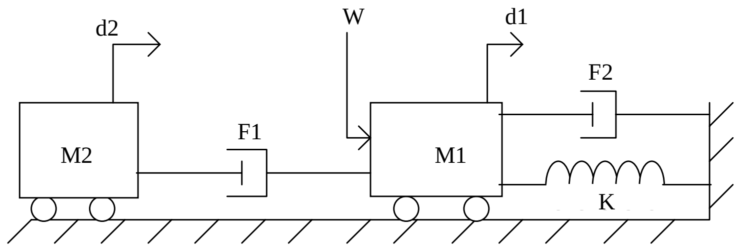
$$\dot{x} = \begin{bmatrix} \dot{V}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -3/2 & -3 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} w(t), x(0) = \begin{bmatrix} 6.67 \\ 6.67 \end{bmatrix}$$

b) English 101 Essay question: Briefly compare and contrast open-loop control versus closed-loop control

Solution: i A closed-loop control utilizes feedback, that is process of returning a signal which occurs somewhere downstream in a process or system back to a point in the system before it was generated. Thus, the input which drives the system is a function of the output or states of the system and can therefore adapt to disturbances, etc.

An open-loop control implies that the input (control) is not a function of the output of the system (i.e., there is no *feedback* loop from output to input and thus the system is unable to adapt to any changes or disturbances which can be measured in the system output. However, open-loop control is generally much cheaper to implement as no sensors at the system output are needed nor comparators or subtractors at the system input.

2a) Write the state variable model for the following mechanical circuit



(Hint: Let $x = \begin{bmatrix} \dot{d}_1 & \dot{d}_2 & K(d_1) \end{bmatrix}^T$ where T means transpose)

<u>Solution:</u> Applying Newton's law at mass M_1 we find:

$$\sum Forces = ma \Rightarrow w(t) - Kd_1 - F_2\dot{d}_1 - F_1(\dot{d}_1 - \dot{d}_2) = M_1\ddot{d}_1$$

or

$$\ddot{d}_1 = -(F_1 + F_2)/M_1\dot{d}_1 + F_1/M_1\dot{d}_2 - K/M_1 d_1 + 1/M_1 w$$

Similarly, we can apply Newton's law at mass M_2 as follows:

$$\sum Forces = ma \Rightarrow -F_1(\dot{d}_2 - \dot{d}_1) = M_2\ddot{d}_2$$

or

$$\ddot{d}_2 = F_1/M_2\dot{d}_1 - F_1/M_2\dot{d}_2$$

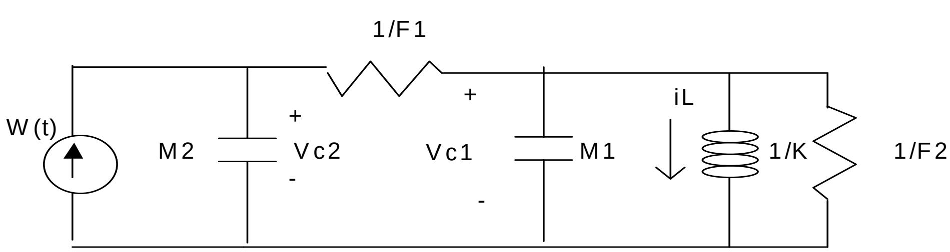
Notice that these two equations involve the time derivatives of our first two state variables, $\dot{d}_1$ and $\dot{d}_2$. To find an equation involving the time derivative of our last state variable, Kd_1, note that (obviously):

$$\frac{d}{dt}(Kd_1) = K(\dot{d}_1).$$

Or, putting all three equations together, we find:

$$\dot{x} = \begin{bmatrix} \ddot{d}_1 \\ \ddot{d}_2 \\ K(\dot{d}_1) \end{bmatrix} = \begin{bmatrix} -(F_1 + F_2)/M_1 & F_1/M_1 & -1/M_1 \\ F_1/M_2 & -F_1/M_2 & 0 \\ K & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ K(d_1) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_2 \\ 0 \end{bmatrix} w$$

b) Now write the state variable model for the following electrical circuit



(Hint: let $x = \begin{bmatrix} Vc_1 & Vc_2 & i_L \end{bmatrix}^T$)

<u>Solution:</u> writing a KCL at the top rightmost node, produces:

$$-w(t) + F_1(Vc_1 - Vc_2) + M_2\dot{V}c_1 = 0$$

or

$$\dot{V}c_1 = -F_1/M_1 Vc_1 + F_1/M_1 Vc_2 + 1/M_1 w$$

Similarly, we can apply a KCL at the top left node to find:

$$F_1(Vc_1 - Vc_2) + M_1\dot{V}c_1 + i_L + F_2Vc_1 = 0$$

or

$$\dot{V}c_2 = (F_2 + F_1)/M_1Vc_1 - F_1/M_1Vc_2 - 1/M_1i_L$$

For our last equation, let's write a KVL involving the inductor voltage:

$$1/K\frac{d}{dt}(i_L) = Vc_1).$$

Or, putting all three equations together, we find:

$$\dot{x} = \begin{bmatrix} \dot{V}c_1 \\ \dot{V}c_2 \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -(F_1+F_2)/M_1 & F_1/M_1 & -1/M_1 \\ F_1/M_2 & -F_1/M_2 & 0 \\ K & 0 & 0 \end{bmatrix} \begin{bmatrix} Vc_1 \\ Vc_2 \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_2 \\ 0 \end{bmatrix} w$$

c) Do you see any similarities between your answers for part 2a) and 2b)?

Solution: They are the exact same equation! How can a mechanical and electrical circuit have the same model?! We'll find out on next lecture!