EE571 - Solution to HW#19

1. a) Use all 10 rules to plot the root locus of $GH=K/(s^2(s+3)(s+8))$

10 rules:1. Starts @ open loop poles @ K=0 which are s=0,0,-3,-8

2. Ends @ open loop zeroes @ K=∞ which are s=∞,∞,∞,∞

3. # of branches = n which is 4

4. R.L. is symmetric WRT σ-axis only

5. ∠ of asymptotes = 180°(odd #)/(n-m) which is 180°(odd #)/(4-0) = 45°,-45°,135°,-135°

6. asymptotes are centered at σ = (b-a)/(n-m) = (0-11)/(4-0) = -11/4 =-2.75

7. R.L. exists at a pt. on σ-axis between -3 ≤ s ≤ -8

8. To find angle of departure from two poles at s=0: Let θ be the angle to the point s1 ≅ 0. Since this point is on the R.L., the angle condition must be satisfied. Thus, $\angle GH(s1) = \angle K/(s^2(s+3)(s+8))\big|_{s=s1}$ = 0° - (θ+θ+0°+0°) = 180° × odd number. Therefore, the angle of departure is θ=90 and -90°.

9. Use Routh array fo find pt. where R.L. crosses jω-axis Char. Eqn: $1+GH = 0 = s^4+11s^3+24s^2+0s+K$. Routh Array looks like:

s^4	1	24	K
s^3	11	0	
s^2	24	K	
s^1	-11K/24		
s^0	K		

Therefore root locus crosses at K=0 and roots are s=0,0,-3,-6. So, root locus crosses at s=0 and s=0.

10. Break-away and break-in pts. @ ∂K/∂s = 0 -> $K=-(s^4+11s^3+24s^2)$. Therefore, ∂K/∂s = 0 = $4s^3+33s^2+48s=0$. The possible beak-away/break-in points are s=0, -6.36, -1.88. Therefore, the two breakaway points are s=0 and s=-6.36 since the point s=-1.88 is not on the root locus but is on the complementary root locus!
(See plot below)

1b) Use all 10 rules to plot the root locus of $GH=K(s+1)/(s^2(s+3)(s+8))$

10 rules:1. Starts @ open loop poles @ K=0 which are s=0,0,-3,-8

2. Ends @ open loop zeroes @ K=∞ which are s=-1,∞,∞,∞

3. # of branches = n which is 4

4. R.L. is symmetric WRT σ-axis only

5. ∠ of asymptotes = 180°(odd #)/(n-m) which is 180°(odd #)/(4-1) = 60°,-60°,180°

6. asymptotes are centered at σ = (b-a)/(n-m) = (1-11)/(4-1) = -10/3 =-3.33

7. R.L. exists at a pt. on σ-axis between -1 ≤ s ≤ -3 and for s ≤ -8

8. To find angle of departure from two poles at s=0: Let θ be the angle to the point s1 ≅ 0. Since this point is on the R.L., the angle condition must be satisfied. Thus, $\angle GH(s1) = \angle K(s+1)/(s^2(s+3)(s+8))\big|_{s=s1}$ = 0° - (θ+θ+0°+0°) = 180° × odd number. Therefore, the angle of departure is θ=90 and -90°.

9. Use Routh array fo find pt. where R.L. crosses jω-axis Char. Eqn: $1+GH = 0 = s^4+11s^3+24s^2+Ks+K$. Routh Array looks like:

s^4	1	24	K
s^3	11	K	
s^2	24-K/11	K	
s^1	K-11K/(24-K/11)		
s^0	K		

Therefore root locus crosses at K=0, K=143. For K=0, the roots are s=0,0,-3,-6. For K=143, the roots are s= -9.89, j3.61, -j3.61, -1.11. Therefore, the root locus crosses at s=0 and s=±j3.61

10. Break-away and break-in pts. @ ∂K/∂s = 0 -> $K=-(s^4+11s^3+24s^2)/(s+1)$. Therefore, ∂K/∂s = 0 = $(4s^3+33s^2+48s)(s+1)- (s^4+11s^3+24s^2)= 3s^4+26s^3+57s^2+48s=0$. The possible beak-away/break-in points are s=0, -5.9, -1.38 + j0.90, -1.38 - j0.90. Therefore, only s=0 is on the root locus and is the only valid breakaway point! (see plots below)

2. a) Use all 10 rules to plot the root locus of $GH=K(s^2+6s+18)/(s(s+1)(s+8)^2)$

10 rules:1. Starts @ open loop poles @ K=0 which are s=0,-1,-8,-8

2. Ends @ open loop zeroes @ K=∞ which are s=-3+j3,-3-j3,∞,∞

3. # of branches = n which is 4

4. R.L. is symmetric WRT σ-axis only

5. ∠ of asymptotes = 180°(odd #)/(n-m) which is 180°(odd #)/(4-2) = 90°,-90°

6. asymptotes are centered at σ = (b-a)/(n-m) = (6-17)/(4-2) = -11/2 =-5.5

7. R.L. exists at a pt. on σ-axis between 0 ≤ s ≤ -1

8. To find angle of departure from two poles at s=-6: Let θ be the angle to the point s1 ≅ -6. Since this point is on the R.L., the angle condition must be satisfied. Thus, $\angle GH(s1) = \angle K(s^2+6s+18)/(s(s+1)(s+8)^2)\big|_{s=s1}$ = (0° - (θ+θ+180°+180°) = 180° × odd number. Therefore, the angle of departure is θ=90 and -90°. To find angle of **arrival** to the zero at s=-3+j3: Let θ be the angle to the point s1 ≅ -3+j3. Since this point is on the R.L., the angle condition must be satisfied. Thus, $\angle GH(s1) = \angle K(s^2+6s+18)/(s(s+1)(s+8)^2)\big|_{s=s1}$ = (θ+90°) - (135°+123.7°+30.96°+30.96°) = 180° × odd number. Therefore, the angle of **arrival** is is θ=50.63° (By symmetry, the angle of arrival to the zero at s=-3-j3 is -50.63°)

9. Use Routh array fo find pt. where R.L. crosses jω-axis. Char. Eqn: $1+GH = 0 = s^4+17s^3+(80+K)s^2+(64+6K)s+18K$. Routh Array looks like:

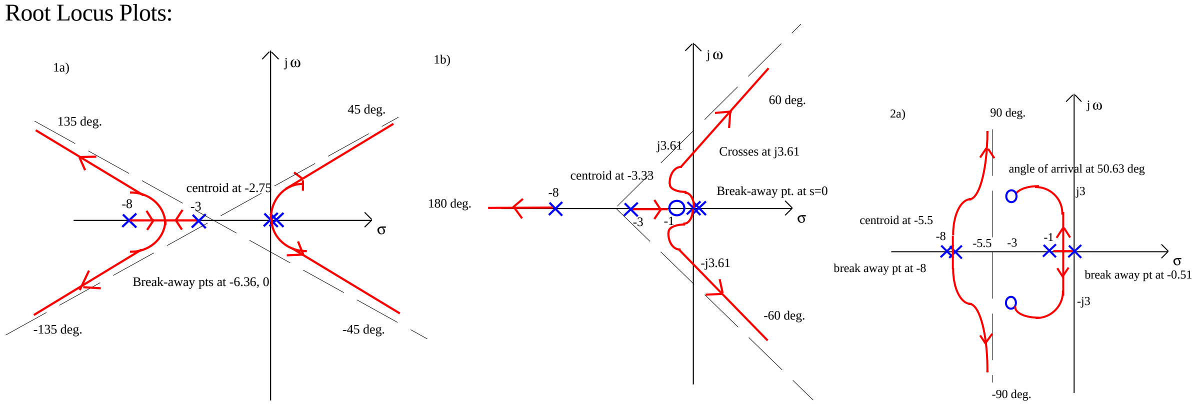
s^4	1	80+K	18K
s^3	17	64+6K	
s^2	80+K-(64+6K)/17	18K	
s^1	64+6K-306K/(80+K-(64+6K)/17)		
s^0	18K		

Therefore system is stable for all K > 0 and root locus crosses at K=0. The roots for K=0 are s=0,-1,-6,-6. Therefore, the root locus intersects the jω axis at s=0 and is in the LHP for all other K > 0.

10. Break-away and break-in pts. @ ∂K/∂s = 0 -> $K= -(s^4+17s^3+80s^2+64s)/(s^2+6s+18)$. Therefore, ∂K/∂s = 0 = $(4s^3+51s^2+160s+64)(s^2+6s+18) - (2s+6)(s^4+17s^3+80s^2+64s)= 2s^5+35s^4+276s^3+1334s^2+2880s+1152=0$. The possible beak-away/break-in points are s=-2.62 + j5.56, -2.62 - j5.56, -8.0, -3.76, -0.51.

Therefore, out of these, only s=-8 and s=-0.51 are on the root locus and are the only valid breakaway points! (see plots below)

Root Locus Plots:



2b) SKETCH!!! (do not use all 10 rules) the form of the root locus for the following open-loop transfer functions: