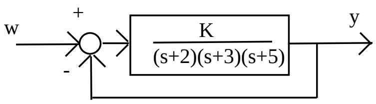
EE571 - Solution to HW#18

1. a) System:

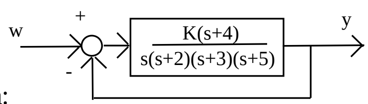


Char. Equation: 1+GH = 0 = $s^3+10s^2+31s+(30+K)$. Our Routh Array looks like:

s^3	1	31
s^2	10	$30+K$
s^1	$\frac{10\cdot 31-1\cdot(30+K)}{10}$	0
s^0	$30+K$	

Thus, to be stable the Routh Array cannot have any sign changes in the first column. Since the first element (1) is positive, all entries must be positive. This implies that -30 < K < 280 for closed-loop stability (i.e., no RHP poles) (if we check using Matlab, we find that for k=-30, the closed-loop poles are {0, -5+j2.45, -5-j2.45}. For K=280, the closed-loop poles are {-10, j5.57, -j5.57}).

1. a) System:

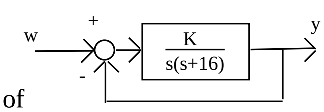


Char. Eqn: 1+GH = 0 = $s^4+10s^3+31s^2+(30+K)s+4K$. Our Routh Array looks like:

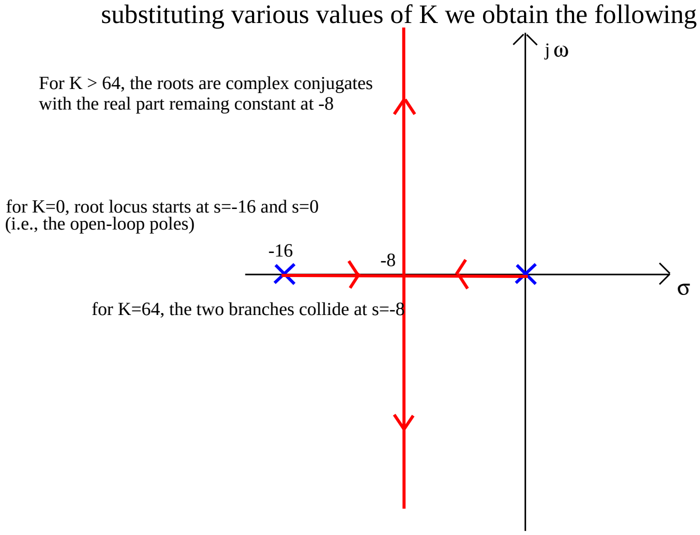
s^4	1	31	$4K$
s^3	10	$30+K$	
s^2	$\frac{10\cdot 31-1\cdot(30+K)}{10}$	$4K$	
s^1	$30+K-\frac{40K}{\frac{10\cdot 31-1\cdot(30+K)}{10}}$		
s^0	$4K$		

Thus, to be stable the Routh Array cannot have any sign changes in the first column. Since the first element (1) is positive, all entries must be positive. The last element implies K > 0. The second to last element implies that (30+K)(280-K))-400K > 0 or $-K^2$ - 150K + 8400 > 0 or K^2 +150K - 8400 < 0 or (K+193.43)(K-43.43) < 0 which implies that -193.43 < K < 43.43. Finally, the third to last element in the Routh Array implies that 280-K > 0 or K < 280. Combing all three conditions produces the following range on K for closed-loop stability: 0 < K < 43.43. (if we check using Matlab, we find that for k=-0, the closed-loop poles are {0, -2, -3 -5}. For K=43.43, the closed-loop poles are {-6.16, -3.84, j2.71, -j2.71}).

c) Sketch the root locus of



. 1+GH = s^2+16s+K = 0 which has roots $s=-8\pm\sqrt{64-K}$. By substituting various values of K we obtain the following root locus plot:



d) Use the angle condition to determine if the following points lie on your root-locus in part c):

i) s=-4+j0: The angle of GH(-4+j0) = angle of 1/((-4)(12)) = 0 - (180 + 0) = -180 degrees. Therefore, the point is on root locus.
ii) s=4+j0: The angle of GH(4+j0) = angle of 1/((4)(20)) = 0 - ((0) + (0)) = 0 degrees. Therefore, the point is **NOT** on the root locus.

iii) s= -8+j2: The angle of GH(-8+j2) = angle of 1/((-8+j2)(8+j2)) = 0 - ((166) + (14)) = -180 degrees. Therefore, the point is on the root locus.

iv) s=-8.5+j2: The angle of GH(-8.5+j2) = angle of 1/((-8.5+j2)(7.5+j2)) = 0 - ((166.8) +(14.9)) = 181.7 degrees. Therefore, the point is **NOT** on the root locus.

e) For those points which lie on the root locus in part d), use the magnitude condition to find the value of K which produces these closed-loop system poles

i) s=-4+j0: The magnitude GH(-4+j0) = magnitude of 1/((-4)(12)) = 1/48. Therefore, the value of K = 48.

iii) s= -8+j2: The magnitude of GH(-8+j2) = magnitude of 1/((-8+j2)(8+j2)) = 1/68. Therefore, the value of K = 68.