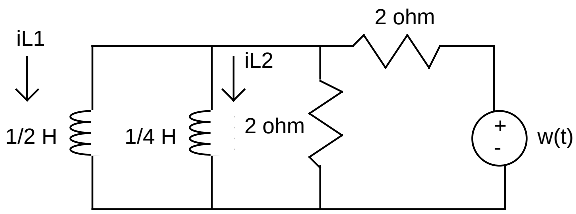
EE571 Solution to HW#12



1a)

Let the state variables be iL1 and iL2. Therefore,

$$\dot{x} = \begin{bmatrix} vL1/L1 \\ vL2/L2 \end{bmatrix} = \begin{bmatrix} 2(vL1) \\ 4(vL2) \end{bmatrix} = \begin{bmatrix} 2(-iL1 - iL2 + 0.5w) \\ 4(-iL1 - iL2 + 0.5w) \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} w$$

Next, to check controllability, let's evaluate the controllability matrix, M=[b Ab … A^{n-1}b]:

$M = \begin{bmatrix} b & Ab \end{bmatrix} = \begin{bmatrix} 1 & -6 \\ 2 & -12 \end{bmatrix}$ which has a determinant of zero! Thus, M has no inverse and the circuit is **NOT** completely controllable.

1b) We need to show that $P^{-1}AP$ has the same eigenvalues as A. The eigenvalues of $P^{-1}AP$ are the roots of det[sI-P^{-1}AP] = det[s P^{-1}P-P^{-1}AP]= det[P^{-1}(sI-A)P] = det[P^{-1}]det[sI-A]det[P] = det[P^{-1}P]det[sI-A] = det[sI-A]. Thus, P^{-1}AP and A have the same characteristic equation and, therefore, have the same eigenvalues.

1c) In the transformed coordinates, the transfer function is H(s)=CP[sI-P^{-1}AP]$^{-1}$$P^{-1}$B = CP[s$P^{-1}$P - P^{-1}AP]$^{-1}$$P^{-1}$B = CP[$P^{-1}$(sI-A)P]$^{-1}$$P^{-1}$B = CP$P^{-1}$(sI-A)$^{-1}PP^{-1}$B = C(sI-A)$^{-1}$B.

2a) The following is the diary file from Matlab:

```
» » A = [-2 -8 -8 -8;-24 -2 -24 -24;12 4 17 13;12 4 13 17]

A =

    -2    -8    -8    -8
   -24    -2   -24   -24
    12     4    17    13
    12     4    13    17

» B=[0;0;3;0]

B =

     0
     0
     3
     0

» M=[B A*B A^2*B A^3*B]

M =

           0         -24         -96        -672
           0         -72       -1440      -31968
           3          51         798       16404
           0          39         750       16212

» det(M)

ans =

   -26873856

» %2a) Since det(M) does not equal zero, the system is completely controllable
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2a) the determinant of M is non-zero, therefore the system is controllable

2b) First, let us find the characteristic equation of the original A matrix:

```
» %2b) Use the poly() command to find the char. eqn. of A:
» poly(A)

ans =

  1.0e+003 *

    0.0010   -0.0300    0.1800   -0.0400   -1.0560

» %Now, plug the negative of these coefficents in for the last row of Apv:
» Apv=[0 1 0 0;0 0 1 0;0 0 0 1;-ans(5) -ans(4) -ans(3) -ans(2)]

Apv =

  1.0e+003 *

         0    0.0010         0         0
         0         0    0.0010         0
         0         0         0    0.0010
    1.0560    0.0400   -0.1800    0.0300

» Bpv=[0;0;0;1]

Bpv =

     0
     0
     0
     1

» Mpv=[Bpv Apv*Bpv Apv^2*Bpv Apv^3*Bpv]

Mpv =

  1.0e+004 *

         0         0         0    0.0001
         0         0    0.0001    0.0030
         0    0.0001    0.0030    0.0720
    0.0001    0.0030    0.0720    1.6240

» Tpv=M*inv(Mpv)

Tpv =

  1.0e+003 *

   -2.1120    0.6240   -0.0240         0
   -1.7280    0.7200   -0.0720         0
    1.5240   -0.1920   -0.0390    0.0030
    0.7320   -0.4200    0.0390         0

» %Check that Tpv is correct:
» inv(Tpv)*A*Tpv

ans =

  1.0e+003 *

    0.0000    0.0010    0.0000    0.0000
    0.0000    0.0000    0.0010    0.0000
    0.0000    0.0000    0.0000    0.0010
    1.0560    0.0400   -0.1800    0.0300

» inv(Tpv)*B

ans =

    0.0000
    0.0000
    0.0000
    1.0000


» %2b) zpvdot=Apv*zpv+Bpv*w
» s_desired=[-5 -10 -11 -12]

s_desired =

    -5   -10   -11   -12

» char_eqn_desired=poly(s_desired)

char_eqn_desired =

           1          38         527        3130        6600

» poly(Apv)

ans =

  1.0e+003 *

    0.0010   -0.0300    0.1800   -0.0400   -1.0560

» Kpv0=6600+Apv(4,1)

Kpv0 =
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```
        7656

» Kpv1=3130+Apv(4,2)

Kpv1 =

        3170

» Kpv2=527+Apv(4,3)

Kpv2 =

  347.0000

» Kpv3=38+Apv(4,4)

Kpv3 =

    68

» Kpv=[Kpv0 Kpv1 Kpv2 Kpv3]

Kpv =

  1.0e+003 *

    7.6560    3.1700    0.3470    0.0680


» %Check the eigenvalues of Apv-BpvKpv
» eig(Apv-Bpv*Kpv)

ans =

  -12.0000
  -11.0000
  -10.0000
   -5.0000
```

2c) $w=-K_{pv}\, z_{pv}$ = -[68 347 3170 7656]z_{pv}.

```
» %2d) Feedback control w = - Kpv*inv(Tpv)*x
» K=Kpv*inv(Tpv)

K =

   46.1250  -70.7500   22.6667  -70.6667

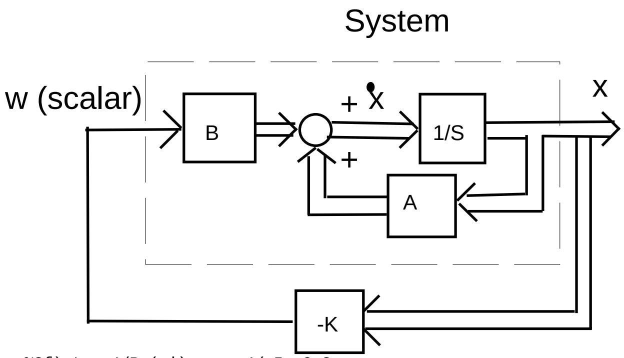
» %Check the eigenvalues of A-BK
» eig(A-B*K)

ans =

  -12.0000
  -11.0000
  -10.0000
   -5.0000
```

2d) In terms of the original states, $w=-K_{pv}\, T_{pv}^{-1}\, x = -Kx$ = -[46.125 -70.75 22.667 -70.667]x.

2e) The block diagram of the closed-loop system is:

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» %2f) ts=-4/Re(si)max= -4/-5 =0.8 sec.
» ts=-4/max(s_desired)

ts =

0.8000
```

2f) The settling time is t_s=-4/Re[s_i]$_{max}$ = -4/(-5) = 0.8 seconds.