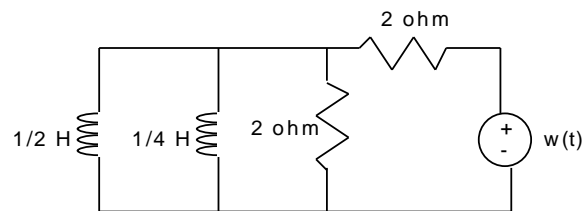


1. a) Start reviewing for Exam I. If you missed a homework, now would be a good time to go back and do it.
- b) Uncontrollable systems often arise when we have a great deal of symmetry in our system. Uncontrollable systems can also occur if we have two energy storage elements which can be combined into one (i.e., in parallel or series). Find the state variable model for the following system then determine whether or not the system is controllable.



- c) Prove that the similarity transformation,  $x=Tz$ , preserves eigenvalues. That is, show that the characteristic equations of  $A$  and  $T^{-1}AT$  are the same. (Hint:  $\det(AB) = \det(A)\det(B)$  and  $\det(T^{-1})=1/\det(T)$ )
- d) Prove that a similarity transformation,  $x=Tz$ , preserves the same transfer function as in the original coordinates (Hint: show that  $CT[sI-T^{-1}AT]^{-1}T^{-1}B+D = C[sI-A]^{-1}B+D$ ) (remember,  $[AB]^{-1} = B^{-1}A^{-1}$  and we can write  $I=T^{-1}T$ )

Please use Matlab on Problem 2!

2. Given the following single-input 4<sup>th</sup> order state variable model,

$$\dot{x} = \begin{bmatrix} -2 & -8 & -8 & -8 \\ -24 & -2 & -24 & -24 \\ 12 & 4 & 17 & 13 \\ 12 & 4 & 13 & 17 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} w$$

- a) Determine the controllability of the system by finding the controllability matrix,  $M$ .
- b) If the system is controllable, find a similarity transformation,  $x=T_{pv}z_{pv}$ , which will put the system into *phase variable* form and write the new state equation for  $z_{pv}$
- c) Next, find a feedback control,  $w=-K_{pv}z_{pv}$  such that the closed-loop phase variable system has the eigenvalues of  $\{-5,-10,-11,-12\}$ .
- d) Given your answer to part c), find an expression for the feedback control  $w$  in terms of the original states  $x$  (hint: recall that  $z_{pv} = T_{pv}^{-1}x$ ).
- e) Draw a block diagram of the closed-loop system.

- f) What is the settling time of the closed-loop system (recall the settling time for a state model is a measure of how fast each the decoupled modes will die out)

Please use Matlab on Problem 2!