EE571 Solution to HW#11

1a) To evaluate the similarity transformation, let's first find the eigenvalues of A:

$$A=\begin{bmatrix}4 & 8\\ 8 & 4\end{bmatrix}\Rightarrow |sI-A|=\begin{vmatrix}s-4 & -8\\ -8 & s-4\end{vmatrix}=s^2-8s-48=(s-12)(s+4)=0 \quad \therefore s_1=12 \ and \ s_2=-4$$

Next, let's find the eigenvectors of A:

$$[s_1I-A]\underline{P}_1=\begin{bmatrix}12-4 & -8\\ -8 & 12-4\end{bmatrix}\begin{bmatrix}p_1\\ p_2\end{bmatrix}=\begin{bmatrix}8 & -8\\ -8 & 8\end{bmatrix}\begin{bmatrix}p_{11}\\ p_{12}\end{bmatrix}=0 \quad \therefore \underline{P}_1=\begin{bmatrix}p_{11}\\ p_{12}\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$[s_2I-A]\underline{P}_2=\begin{bmatrix}-4-4 & -8\\ -8 & -4-4\end{bmatrix}\begin{bmatrix}p_1\\ p_2\end{bmatrix}=\begin{bmatrix}-8 & -8\\ -8 & -8\end{bmatrix}\begin{bmatrix}p_{21}\\ p_{22}\end{bmatrix}=0 \quad \therefore \underline{P}_2=\begin{bmatrix}p_{21}\\ p_{22}\end{bmatrix}=\begin{bmatrix}1\\ -1\end{bmatrix}$$

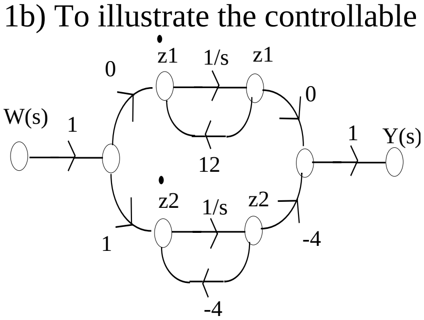
Therefore $P=[\underline{P}_1 \quad \underline{P}_2]=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ and $S=\begin{bmatrix}s_1 & 0\\ 0 & s_2\end{bmatrix}=\begin{bmatrix}12 & 0\\ 0 & -4\end{bmatrix}$. Thus, if we let x=Pz and

$$\dot{z}=P^{-1}APz+P^{-1}bw=\begin{bmatrix}1/2 & 1/2\\ 1/2 & -1/2\end{bmatrix}\begin{bmatrix}4 & 8\\ 8 & 4\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}z+\begin{bmatrix}1/2 & 1/2\\ 1/2 & -1/2\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}w=\begin{bmatrix}12 & 0\\ 0 & -4\end{bmatrix}z+\begin{bmatrix}0\\ 1\end{bmatrix}w$$

The output equation in the decoupled coordinates is

$$y=CPz=[-2 \quad 2]\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}z=[0 \quad -4]z$$

1b) To illustrate the controllable and observable eigenvalues (modes), let's look at the decoupled SFG:

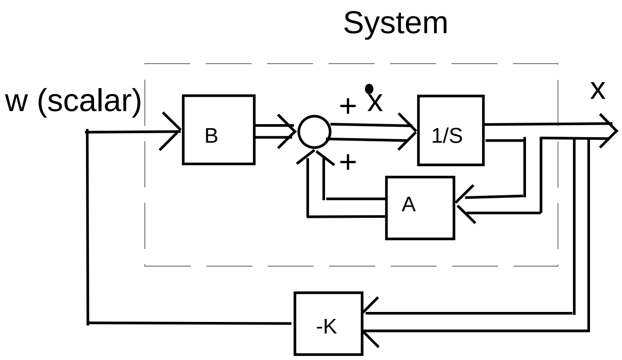


Clearly, $s_1=12$ (or mode z_1) is uncontrollable, unobservable, and unstable. Contrastingly, $s_2=-4$ (or mode z_2) is controllable, observable, and stable.

1d) The system is $\dot{x}=\begin{bmatrix}0 & 1\\ 5 & -4\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}w=\begin{bmatrix}0 & 1\\ -a_0 & -a_1\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}w$ which is in phase variable form. Hence, the characteristic equation is $|sI-A|=s^2+a_1s+a_0=s^2+4s-5=(s+5)(s-1)=0$. Therefore, the eigenvalues are $s_1=-5$ and $s_2=1$. Thus, the system is NOT asymptotically stable and is actually unstable because $s_2=1$ is in the RHP.

1e) Our desired characteristic equation is $(s+5)(s+6)=s^2+11s+30=0$. Let $w=-Kx=-[k_0 \ k_1]x$. Then, the closed-loop system is: $\dot{x}=\begin{bmatrix}0 & 1\\ -(k_0-5) & -(k_1+4)\end{bmatrix}x$. Note, that this is still in phase-variable form! So the closed-loop characteristic equation is solution is $|sI-(A-bK)|=s^2+(k_1+4)s+(k_0-5)=0$. Equating coefficients, we obtain: $(k_1+4)=11$ and $(k_0-5)=30$ thus $k_0=35$ and $k_1=7$. So the feedback control is $w=-[k_0 \ k_1]x=-[35 \ 7]x$.

1f) The vector block diagram is:



1g) The settling time of the new closed-loop system is t_s=-4/-5=0.8 seconds. This is a measure of how quickly the **decoupled** modes will decay to within 2% of their initial value.