EE571 Cheat Sheet II

Root Locus: Let $KGH = K\frac{s^m + bs^{m-1} + ...}{s^n + as^{n-1} + ...}$ be the open-loop transfer function.

Angle Condition: A point s1 is on the R.L. if ∠GH(s1) = 180° x (odd number)

Magnitude Condition: If a point s1 lies on the R.L., then the value of K can be found from |KGH(s1)| = 1

10 rules:
1. Starts @ open loop poles @ K=0
2. Ends @ open loop zeroes @ K=∞
3. # of branches = n
4. R.L. is symmetric WRT σ-axis and any other axis about which the open-loop pole zero configuration is symmetric
5. ∠ of asymptotes = 180° (odd #)/(n-m)
6. asymptotes are centered at σ = (b-a)/(n-m)
7. R.L. exists at a pt. on σ-axis if # of poles and zeroes of GH to the right is odd
8. To find angle of departure - draw a pt. very close to pole or zero and use ∠ condition
9. Use Routh array to find pt. where R.L. crosses jω-axis
10. Break-away and break-in pts. @ ∂K/∂s = 0

Root Locus Compensation: $\angle G_c(s1)$ = angle of deficiency = 180° x (odd #) - $\angle GH|_{\text{desired poles}}$; $G_{PID} = K_i/s + K_p + K_d s$

Error Analysis: (Valid for unity-feedback with closed loop system stable):

$$K_p = \lim_{s\to 0} G(s) \quad K_v = \lim_{s\to 0} sG(s) \quad K_a = \lim_{s\to 0} s^2 G(s) \quad ess|_{step} = 1/(1+K_p) \quad ess|_{ramp} = 1/K_v \quad ess|_{parabola} = 1/K_a$$

Routh Array: Check the RHP roots of $b_n s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + ... + b_1 s + b_0 = 0$

s^n	b_n	b_{n-2}	b_{n-4} ...
s^{n-1}	b_{n-1}	b_{n-3}	b_{n-5} ...
s^{n-2}	c_1	c_2	...
	.		
	:		
s^1			
s^0			

$$c_1 = \frac{b_{n-1}b_{n-2} - b_n b_{n-3}}{b_{n-1}} \qquad c_2 = \frac{b_{n-1}b_{n-4} - b_n b_{n-5}}{b_{n-1}}$$

Frequency Response Methods:

Mapping Theorem: Let F(s) be a ratio of two polynomials in s. Choose a closed contour and map it into the F(s) - plane. Then, the number of encirclements the mapped contour makes about the origin in the F(s)-plane (in the same direction as the original contour), in N=Z-P where Z= # of zeroes of F(s) enclosed by the original contour and P = # of poles of F(s) enclosed by the original contour

Decibel: 20log(|·|)

gain margin = $1/|GH(j\omega_{cp})|$ where $\angle GH(j\omega_{cp}) = -180°$ **phase margin** = $180° + \angle GH(j\omega_{cg})$ where $|GH(j\omega_{cg})| = 1$ or 0 dB

Conversion Factors: $2 + j = \sqrt{5}\angle 26.6^o$; $\sqrt{3} + j = 2\angle 30^o$; $1 + j = \sqrt{2}\angle 45^o$; $1 + j\sqrt{3} = 2\angle 60^o$; $1 + j2 = \sqrt{5}\angle 63.4^o$

1 = 0 dB 2 = 6 dB 3 = 9.5 dB 5 = 14 dB 8 = 18 dB

Lead Compensation

$G_c(s) = K_c \frac{Ts+1}{\alpha Ts+1}$ $0 < \alpha < 1$

Maximum phase:

$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$ occurs at $\omega_m = 1/(\sqrt{\alpha}T)$

Extra gain at $\omega_m = 1/\sqrt{\alpha}$

Lag Compensation

$G_{lag}(s) = K_{lag}\frac{Ts+1}{\beta Ts+1}$ $1 < \beta < \infty$

Find ω_{cgnew} such that

$\angle GH(j\omega_{cg})$ = -180°+ desired pm + fudge factor

Attenuation at high frequencies = $1/\beta$

Sensitivity: $S_y^x = \frac{\partial x}{\partial y}\frac{Y}{X}\Big|_{Xnom,Ynom}$

Observer Equation: $\dot{\hat{x}} = A\hat{x} + Bw + K_o(y - C\hat{x})$