Electronic Circuits Laboratory EE462G

Lab Background,
Lab Safety, Experimental Errors , and Statistics

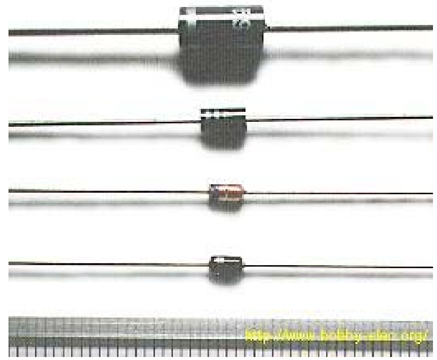
Course Web Link:
http://www.engr.uky.edu/~zhichen/TEACHING/teaching.html

Revised by Dr. Zhi David Chen Fall 2018

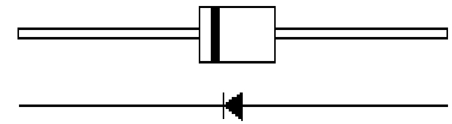
Electronic Components Studied

Diodes

- *pn* junction
- Zener

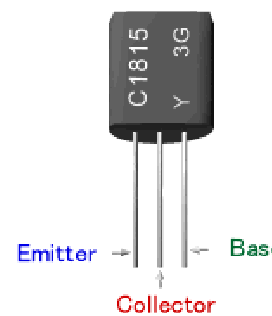


http://www.interq.or.jp/japan/se-inoue/e_diode.htm

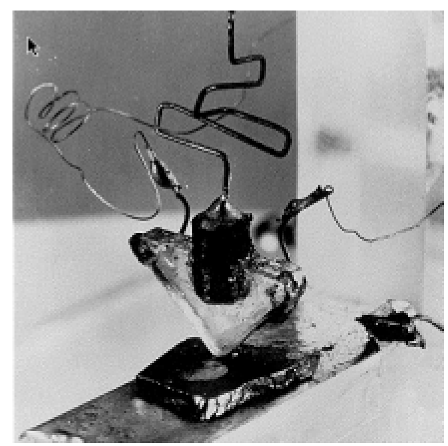


Transistors

- Field Effect Transistors – (FETs)
- Bipolar Junction Transistors – (BJTs)



The first transistor



http://www.mtmi.vu.lt/pfk/funkc_dariniai/transistor/

Laboratory Component Supplies

Parts can be purchased from UK Bookstore

See the EE462G Supply List

Lab Safety

How much electricity does it take to kill or severely injure?

- 0.01 A = Mild sensation, threshold of perception
- 0.1 A = Severe shock, extreme breathing difficulties, cannot let go, painful
- 1.0 A = Severe burns, respiratory paralysis, ventricular fibrillations

Lab Safety

- Turn off power supply and function generator when building or modifying a circuit.
- If power supply is acting strangely or making components very hot, do not use. Disconnect power and bring it to the attention of the TA or lab technician.
- Don’t stick face near circuit while power is on. Eye protection is recommended.
- Be aware of ground loops! Particularly those made with your body.
- Don’t work with wet or moist hands.
- No food or drink is permitted in the lab.

Lab Safety

- Leave the lab table as neat or better than the way you found it. Clean off table top and arrange equipment neatly. Turn off all equipment (log off PCs) before leaving.
- Move equipment using the handle or chassis. Don’t move equipment by pulling on chords or probes.
- Do not open up or take apart equipment.
- If equipment is missing or not working, bring it to the attention of the TA or lab technician.

Lab and Instrumentation Terms

Use these words in your write ups and discussions!

Accuracy

Conformity of measurement to true value.

Error

Difference between measured and true value or comparison value, often normalize by the true or expected value and given in percent.

Precision

Number of significant digits in measurement, depends on both the instrument resolution and the repeatability of the process being measured.

Resolution

Smallest measurable increment (reporting a number beyond the precision or resolution of the instrument is meaningless and misleading; the numbers will not be repeatable).

Lab and Instrumentation Terms

Mean Error

The average error between a series of measured values and the corresponding true value (accuracy/bias):

$$\overline{\Delta} = \frac{1}{N}\sum_{i=1}^{N}\Delta_i = \frac{1}{N}\sum_{i=1}^{N}\left(\mu - x_i\right)$$

x_i is the i^{th} **measurement** out of N independent measurements.

μ is the **true or comparison value**.

$\overline{\Delta}$ is the **mean error** associated with the measurements.

Lab and Instrumentation Terms

Standard Deviation

The average deviation of values from their mean indicates the variability of the measurement (effective precision):

$$S = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\bar{x} - x_i\right)^2}$$

x_i is the i^{th} **measurement** out of N independent measurements.

$\bar{x}$ is the **sample mean** of the measurements

S is the **sample standard deviation.**

Lab and Instrumentation Terms

Percent Absolute Error

Normalized error:

$$E_i = \frac{100(\mu - x_i)}{\mu}$$

If the true values are not known, **you cannot compute error**. Instead you compute a difference between 2 values that have similar meaning. For example, you can compute the differences between the measured values in an experiment and the value predicted by theory. Then treat the value with the least expected error as the true value in order to compare theory and experiment.

Experimental Error

- Conclusions drawn from experimental measurements must be tempered by the error/variability/uncertainty in the measured values.

- "Experimental error" exists in measurements creating uncertainty due to variations from the instrument resolution and measurement repeatability.

- The challenge of good experimental design is to limit sources of variability in the measurements to minimize experimental error relative to the question the experiment is trying to answer.

- Once the experiment is designed and measurements made, the experimental error must be characterized in order to determine the level of uncertainty in the data.

Measurements and Statistics

Experimental measurements were performed where the expected or predicted value was 10.2 Volts. Measurements were made under 3 different conditions. The following numbers were measured and recorded:

- Condition 1: 9.43, 10.5, 11.4, 10.33
- Condition 2: 10.1, 10.2, 10.15, 10.14
- Condition 3: 6.00, 5.72, 8.21, 6.33, 6.57

For which condition(s) are the differences in the measurement and the predicted values most likely due to experimental error.

Confidence Intervals

<u>Confidence Interval Estimation</u>

A confidence interval is a random interval whose end points β and γ are functions of the observed random variables such that the probability of the inequality:

$$\beta < \theta < \gamma \qquad \text{where } \theta \text{ is the true/exact value}$$

is satisfied to some predetermined probability ($1-\alpha$):

$$\Pr[\beta < \theta < \gamma] = 1-\alpha \qquad \text{Confidence level}$$

Typically α = .05 for a 95% confidence interval. ---95% of the measured data are between β and γ!

A useful distribution for mean values estimated from a finite number of samples is the Student's *t* distribution.

Student's *t*-Statistic

<u>Student's *t* Distribution</u>

Let $x_1, x_{2,} x_3, \ldots x_N$ **be normally distributed**

- **true mean equals μ**
- **true standard deviation equals S**

- **sample mean** $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$
- **standard deviation** $S = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}$

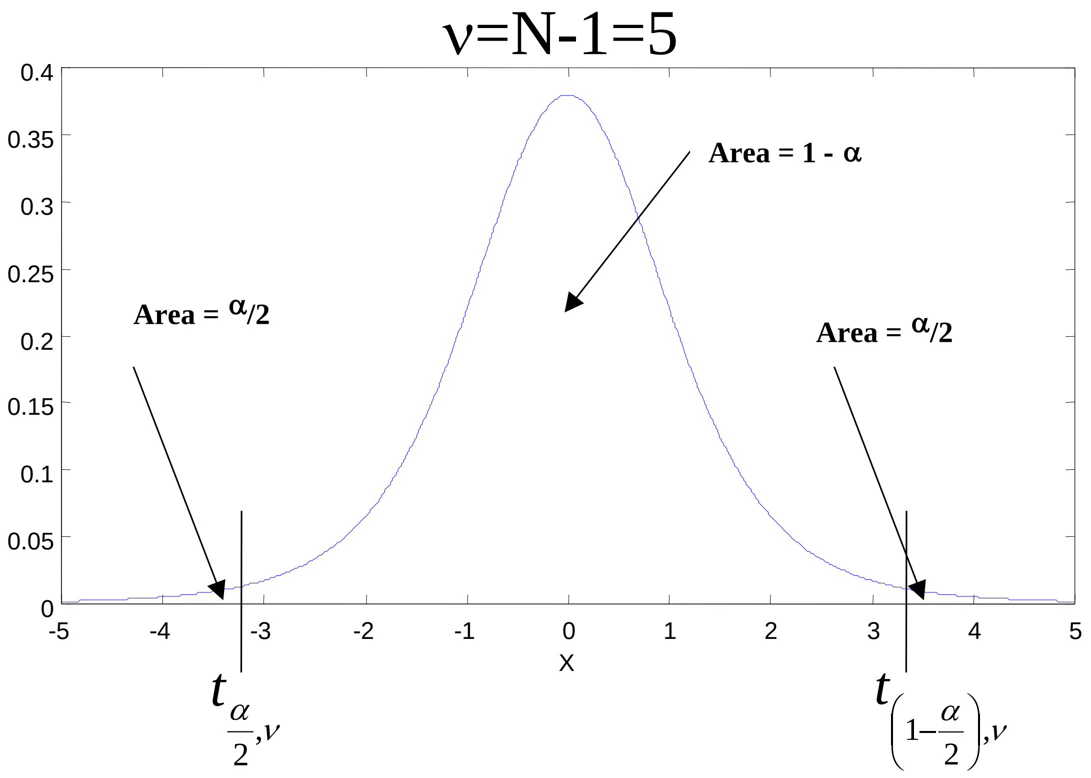
Then the *t*-statistic becomes $t_\nu = \frac{\bar{x} - \mu}{S / \sqrt{N}}$

with $\nu = N\text{-}1$ **degrees of freedom and has the student's *t*-distribution**

Student's t-Statistic

Let random variables $t_{\alpha/2,\nu}$ **and** $t_{1-\alpha/2,\nu}$ **denote values where the area between these points under the density function is:** $P = \left(1-\frac{\alpha}{2}\right) - \left(\frac{\alpha}{2}\right) = 1-\alpha$

Note: For α=.05, the area between t values is 0.95. Here ν is the Degree of Freedom.



Computing Confidence Intervals

Recall the t-statistic was defined as: $$t_v = \frac{\bar{x} - \mu}{S/\sqrt{N}}$$

Therefore, the true mean is in a neighborhood (interval) of the sample mean with probability (confidence) 1-α:

$$\Pr\left[\underbrace{\bar{x} - (t_{1-\alpha/2,\nu})\frac{S}{\sqrt{N}}}_{\text{Lower Confidence Limit}} < \mu < \underbrace{\bar{x} + (t_{1-\alpha/2,\nu})\frac{S}{\sqrt{N}}}_{\text{Upper Confidence Limit}}\right] = 1 - \alpha$$

t-statistic Table $t_{1-\alpha/2,\nu}$

See an online calculator:
http://stattrek.com/online-calculator/t-distribution.aspx

Degrees of freedom	t for a symmetric 95% probability interval	Degrees of freedom	t for a symmetric 95% probability interval
2	4.3027	11	2.2010
3	3.1824	12	2.1788
4	2.7764	13	2.1604
5	2.5706	14	2.1448
6	2.4469	15	2.1314
7	2.3646	16	2.1199
8	2.3060	17	2.1098
9	2.2622	18	2.1009
10	2.2281	19	2.0930

Experimental Error Example

Find the Mean, Standard Deviation, Upper 95% confidence limit, Lower 95% confidence limit. Which conditions suggest the differences between the measured and the predicted values were not significant (@ 95% Level)?

EE462 Measurement Example - Experimental Error				
Measurement	Condition1	Condition2	Condition3	
1	9.43	10.10	6.00	
2	10.50	10.20	5.72	
3	10.00	10.10	8.21	
4	11.40	10.15	6.33	
Mean				
Standard Dev				
Upper 95% confidence limit				
Lower 95% confidence limit				

Solution of the Example

Calculate Mean: $$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Standard Deviation: $$S = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Find the t-score:
If it is 95% confidence limit (α=0.05), you can use the above Table.
For any other α values, use the online t-score calculator (http://stattrek.com/online-calculator/t-distribution.aspx).
Input Degrees of Freedom ν=N-1 and Cumulative Probability=1- (α/2), you obtain t-scores: $t_{1-\alpha/2,\nu}$

Lower (1- α) Confidence Limit: $$\bar{x} - (t_{1-\alpha/2,\nu})\frac{S}{\sqrt{N}}$$

Upper (1- α) Confidence Limit: $$\bar{x} + (t_{1-\alpha/2,\nu})\frac{S}{\sqrt{N}}$$

Results of the Example

Which conditions suggest the differences between the measured and the predicted values were not significant (@ 95% Level).

EE462 Measurement Example - Experimental Error				
Measurement	Condition1	Condition2	Condition3	Comparison Value (theory)
1	9.43	10.10	6.00	10.20
2	10.50	10.20	5.72	T-statistic
3	10.00	10.10	8.21	3.18
4	11.40	10.15	6.33	
Mean	10.33	10.14	6.57	
Standard Dev	0.84	0.05	1.12	
Upper 95% confidence limit	11.66	10.21	8.35	
Lower 95% confidence limit	9.00	10.06	4.78	

Repeating Measurements and Averaging

- If a value is being measured/estimated from a circuit, it should be measured **multiple times**, where sources of variability change **independently** from measurement to measurement, while the property of interest does not.

- The **mean** can be computed from the repeated measurements as an **estimate of the true value,** and the **standard deviation** (or a comparable statistic related to variation) can be computed as an **uncertainty** estimate.

Curve Fitting to Sequence of Data Points

- If a function (or waveform) is of interest, then a sequence of points over the function's domain must be measured. The sequence of domain points must be dense enough to resolve critical features of the function. The function values should be measured **multiple times** with the property of interest held constant.

- The **mean** of the measured function points at each domain point can serve as an **estimate.** The **standard deviation** (or comparable statistic) can be computed as an estimate of the **uncertainty**. Confidence limits can be computed for each function point and included as error bars on the plot.

Parametric Curve Fitting

- If a sequence of measured points is expected to fit a particular curve controlled by a few parameters (such as an exponential, sinusoid, polynomial ….), then the error between the curve and the measured points can be computed as a function of the curve's parameters. The parameter set yielding the minimum error can then be used as the "best-fit" parameter estimate for describing the data.
- The error between a function and a set of points is often described by the mean square error (MSE):

$$\varepsilon(\alpha) = \frac{1}{N}\sum_{i=1}^{N}\left(f(x_i;\alpha) - \tilde{V}_i\right)^2$$

$\tilde{V}_i$ is measured data corresponding to domain point x_i

$f(x_i;\alpha)$ is the function intended to fit the measured points with parameter α

Curve Fit Example

- Assume a series of voltage outputs in response to a step input follows and exponential response of the form:

$$v_o(t) = \left(1 - \exp(-\alpha t)\right)$$

- An experiment was performed where the system was driven with a step and the outputs were measured at times .01, .02, .03, .04, and .05 seconds. For 5 independent trials the corresponding outputs were:

time	t=.01	t=.02	t=.03	t=.04	t=.05
Trial 1	0.5955	0.8719	0.8937	1.0293	0.8277
Trial 2	0.7014	0.8800	0.9685	1.0666	0.9740
Trial 3	0.6228	0.7757	0.8338	0.9324	1.0129
Trial 4	0.5942	0.7713	1.0253	0.9318	0.9925
Trial 5	0.6321	0.8472	0.9780	0.9632	0.8768

- Write a Matlab script to find the best value of α for each trial. Then find the mean, standard deviation and 95% confidence limits for the α estimates over all the independent trials.
- Plot the parametric curves based on the mean α and those corresponding to the 95% confidence limits.
- Plot the measured data points on the same graph and compare them relative to the 95% confidence intervals.
- Comment on the confidence limits for α in relationship to the true value (which was 100 in this case).

Curve Fit Example

- Sample Matlab script to solve problem and then some extra:

```
%  Create vector corresponding to time points for each measurement
t = [.01, .02, .03, .04, .05];

%  Type in measured data points where each column corresponds to the
%  time point and each row corresponds to a trial.
expmst = [0.5955    0.8719    0.8937    1.0293    0.8277; ...
    0.7014    0.8800    0.9685    1.0666    0.9740; ...
    0.6228    0.7757    0.8338    0.9324    1.0129; ...
    0.5942    0.7713    1.0253    0.9318    0.9925; ...
    0.6321    0.8472    0.9780    0.9632    0.8768];

%  Type in a range of alpha values to try in the curve fit process
a = [50:.02:130];  %  Vector from 50 to 130 in steps of .02
```

Curve Fit Example

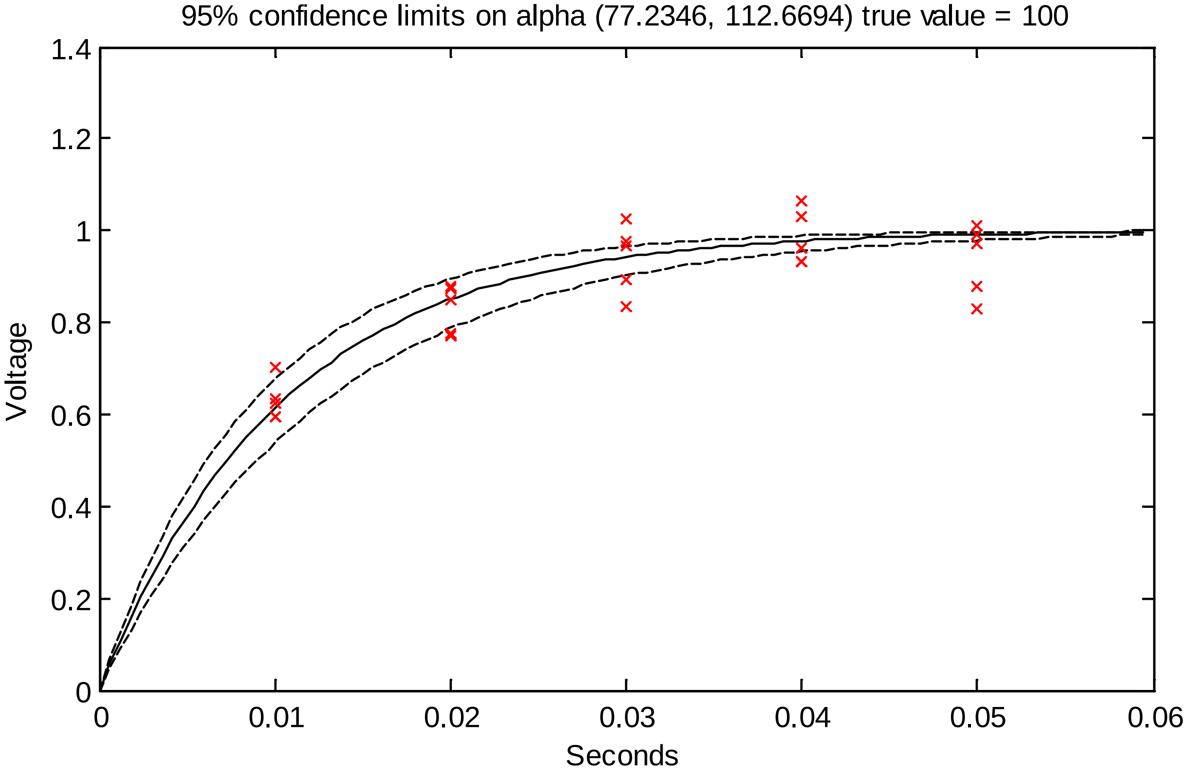
```
% Loop to estimate alpha for each trial
for kt=1:5 % (kt is the index for each experiment/measurement set)
   %  Loop to compute the MSE for each alpha
   for ka=1:length(a)
      %  MSE between data and parametric function is trial alpha value
       mse(ka) = mean((expmst(kt,:) - (1-exp(-a(ka)*t))).^2);
   end
   %  Find minimum MSE and corresponding alpha for each measurement set
   [tmpval, tmppos] = min(mse);
   aest(kt) = a(tmppos(1));  %  if several points tied for minimum value, take only first one
   mseaest(kt) = tmpval(1);
end
%  Compute statistics on alpha estimates
mnalph = mean(aest);  %  Mean
stdalph = std(aest);  %  Standard deviation
ts = tinv(1-.05/2,5-1); %  t statistic value for 95% confidence interval
culim = mnalph + ts*stdalph/sqrt(5);  %  Upper limit for 95% confidence interval
cllim = mnalph - ts*stdalph/sqrt(5);  %  Lower limit for 95% confidence interval
```

Curve Fit Example

```
%   Plot statistics
tax = .06*[0:100-1]/100;     %  Compute a denser time point axis for plotting parametric curve
mcrv = 1-exp(-mnalph*tax);   %  Curve with mean alpha
ucrv = 1-exp(-culim*tax);    %  Curve with upper confidence limit alpha
lcrv = 1-exp(-cllim*tax);    %  Curve with lower confidence limit alpha
%  Plot mean curve
plot(tax,mcrv,'k')
hold on   %  Hold the current plot and add the following:
plot(tax,ucrv,'k--')   %  Upper confidence limit alpha with broken line
plot(tax,lcrv,'k--')   %  Lower confidence limit alpha with broken line
%   Loop through all rows of data matrix and place actual data points
%   on plot as red x's
for kt=1:5
   plot(t,expmst(kt,:),'xr')
end
hold off   %  Release plot
% label plots and display confidence limits
xlabel('Seconds')
ylabel('Voltage')
title(['95% confidence limits on \alpha (' num2str(cllim) ', ' num2str(culim) ') true value = 100'])
```

Curve Fit Example

Resulting plot and confidence limits, data was simulated with a true value for alpha of 100.



If this experiment was run 100 times how often would you expect the confidence limits to include the true value?

Homework #1

Download the PDF tutorial document entitled “The XYZs of Oscilloscopes” (http://www.engr.uky.edu/~zhichen/TEACHING/teaching.html) and read through the tutorial on oscilloscopes (~50 Pages).

1. Print out pages 51 through 55.
2. Put your name on the first page and complete the written exercises.
3. Use the answer key to determine number of wrong responses, put that number on the front page next to your name and circle it.
4. On a separate sheet of paper, briefly describe (10 to 15 sentences) how an analog oscilloscope displays a periodic waveform. Assume the trigger is set to the same channel as the periodic waveform being displayed. Be sure to clearly describe the relationship between the trigger, oscillating signal, and CRT sweep rate. The reader should have a good idea why a periodic waveform appears stationary on the oscilloscope. You can sketch diagrams if that helps your explanation.

Attach all pages together and hand in at the beginning of the next lecture. The assignment grade will depend on completing 1 through 3 and the quality of the explanation in 4.

HW #2

1. Write a Matlab function with an input being a vector of data points (independent measurements) and the outputs being the mean value along with the upper and lower 95% confidence limits. *Please use equations in the "Solution of Example" slide.* The first line of your code defines the function syntax and should look like:

   function [mv, uc, lc] = confid(x)

   mv - mean value; uc – upper confidence limit; lc – lower confidence limit

   Hand in a commented hardcopy of the code. The first set of comments (right after the function declaration) should explain how the function is used and what the input and output arguments are. Then every line should have a comment that explains its purpose. Test your function on the following vector of measurements:

   x = [19.1, 22.2, 16.9, 19.4, 20.2, 18.0, 21.1]

   write the resulting mean value with the upper and lower confidence limits on a hardcopy print out of the code and hand in.

HW #2 (cont'd)

2. Assume the impulse response of your circuit has the form:

$$v_o(t) = \exp\left(-\alpha t^2\right)u(t)$$

where t is time in seconds, α is a function parameter controlling the roll-off over time, and $u(t)$ is the unit step function.

Hint: If t is a vector of time points in Matlab, the vector of parametric function values corresponding to points in t is written as v = exp(-alph*t.^2)

For 5 trials and measurements at time points .01, .02, .03, .04, and .05 seconds, the data is given as (unit of volts)

	t=0.01	t=.02	t=.03	t=0.04	t=0.05	seconds
Trial 1	0.9268	0.7855	0.5270	0.3029	0.1731	volts
Trial 2	0.9909	0.8888	0.5671	0.3384	0.2305	volts
Trial 3	0.9124	0.8633	0.6109	0.2904	0.1619	volts
Trial 4	1.0038	0.7895	0.5753	0.2664	0.2068	volts
Trial 5	0.9711	0.7383	0.5055	0.3425	0.1427	volts

Write a Matlab script to estimate the value of α through a curve fit program for each trial (Hint the true value of sigma is somewhere between 100 and 1000). Compute the mean of the resulting α's, and find the 95% confidence limits (just as in the example). Hand in a hard copy of the commented script with the resulting mean α and its confidence limits and also provide a graph along with the code.

Matlab Resources

- The help files in Matlab, go to Demos tab for examples
- In Matlab type *help function*
- In Matlab type *help script*
- Google *Matlab tutorial* or *Matlab basics*

- See Matlab Files and Description in the course webpage:
  http://www.engr.uky.edu/~zhichen/TEACHING/teaching.html

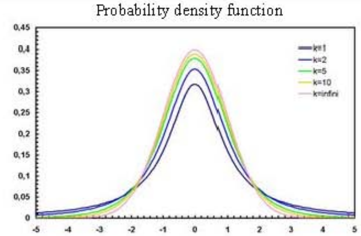
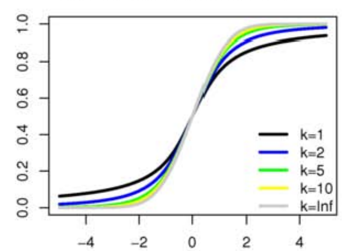
- Manual on Matlab Basics
  http://www.mathworks.com/academia/student_version/doc_r14.html
  Download PDF on “Learning Matlab”

- MATLAB Tutorials:
  http://www.mathworks.com/academia/student_center/tutorials/index.html
  A graphic description to step through basic exercises in Matlab.

Student's t-distribution

From Wikipedia, the free encyclopedia

Student's *t*

Probability density function	
	
Cumulative distribution function	
	
Parameters	$\nu > 0$ deg. of freedom (real)
Support	$x \in (-\infty; +\infty)$
Probability density function (pdf)	$\frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\,\Gamma(\nu/2)}(1+x^2/\nu)^{-(\nu+1)/2}$
Cumulative distribution function (cdf)	$\frac{1}{2} + \frac{x\Gamma\left((\nu+1)/2\right)\,{}_2F_1\left(\frac{1}{2},(\nu+1)/2;\frac{3}{2};-\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\,\Gamma(\nu/2)}$ where ${}_2F_1$ is the hypergeometric function
Mean	0 for $\nu > 1$, otherwise undefined
Median	0
Mode	0
Variance	$\frac{\nu}{\nu-2}$ for $\nu > 2$, otherwise undefined
Skewness	0 for $\nu > 3$
Excess kurtosis	$\frac{6}{\nu-4}$ for $\nu > 4$
Entropy	

In probability and statistics, the ***t*-distribution** or **Student's *t*-distribution** is a probability distribution that arises in the problem of estimating the mean of a normally distributed population when the sample size is small. It is the basis of the popular Student's *t*-tests for the statistical significance of the difference between two sample means, and for confidence intervals for the difference between two population means. The **Student's *t*-distribution** is a special case of the generalised hyperbolic distribution.

The derivation of the *t*-distribution was first published in 1908 by William Sealy Gosset, while he worked at a Guinness Brewery in Dublin. He was not allowed to publish under his own name, so the paper was written under the pseudonym ***Student***. The *t*-test and the associated theory became well-known through the work of R.A. Fisher, who called the distribution "Student's distribution".

Student's distribution arises when (as in nearly all practical statistical work) the population standard deviation is unknown and has to be estimated from the data. Textbook problems treating the standard deviation as if it were known are of two kinds: (1) those in which the sample size is so large that one may treat a data-based estimate of the variance as if it were certain, and (2) those that illustrate mathematical reasoning, in which the problem of estimating the standard deviation is temporarily ignored because that is not the point that the author or instructor is then explaining.

	$\frac{\nu+1}{2}\left[\psi(\frac{1+\nu}{2})-\psi(\frac{\nu}{2})\right]$ $+\log\left[\sqrt{\nu}B(\frac{\nu}{2},\frac{1}{2})\right]$ ▪ ψ: digamma function, ▪ B: beta function
Moment-generating function (mgf)	(Not defined)
Characteristic function	

Contents

Why Student's *t*-distribution

Confidence intervals and hypothesis tests rely on Student's *t*-distribution to cope with uncertainty resulting from estimating the standard deviation from a sample, whereas if the population standard deviation were known, a normal distribution would be used.

How Student's *t*-distribution comes about

Suppose $X_1, ..., X_n$ are independent random variables that are normally distributed with expected value μ and variance σ^2. Let

$$\overline{X}_n = (X_1 + \cdots + X_n)/n$$

be the sample mean, and

$$S_n{}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)^2$$

be the sample variance. It is readily shown that the quantity

$$Z = \frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}}$$